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### Financial Fragility and the Fiscal Multiplier

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# Financial Fragility and the Fiscal Multiplier

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## Abstract

We investigate the effectiveness of ‘Keynesian’ fiscal stimuli when government deficits and debt rollovers are (possibly partially) financed by balance sheet constrained financial intermediaries. Because financial intermediaries operate under a leverage constraint, deficit financing of fiscal stimulus packages will cause interest rates to rise as private loans are crowded out by government debt in the credit provision channel. This lowers investment and (future) capital stocks, which affects output negatively for a prolonged period. Anticipations of these future consequences cause the price of capital and bonds to drop immediately when the policy is announced, inflicting capital losses on banks which leads to further tightening of leverage constraints and credit market conditions. This balance sheet effect triggers a negative amplification cycle further lowering the fiscal multiplier. Longer maturity debt leads to larger capital losses and lower Keynesian multipliers. When in addition sovereign default risk is introduced, additional capital losses may occur and outcomes deteriorate further after a deficit financed stimulus package, eventually implying a cumulative Keynesian multiplier close to zero or even negative. We do not argue that multipliers are always negative; but financial fragility and sovereign risk problems may severely lower them, possibly to the point of becoming negative. Our model indicates that calls from European politicians to engage in deficit-financed stimulus to prop up economic growth, might backfire in an environment characterised by weak banks and weak public finances. We show that to restore effectiveness to fiscal policy we need an accommodating monetary policy stance and well capitalized banks.

*Keywords:* ‘Financial Intermediation; Macrofinancial Fragility; Fiscal Policy; Sovereign Default Risk’

*JEL classification:* E44; E62; H30

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We like to thank Keith Kuester from University of Bonn for sharing data on the European stresstest of the financial sector.

# 1 Introduction

The Great Recession of 2008-2009 has sparked a renewed debate among economists on the costs and benefits of fiscal stimulus. Driven by crumbling financial markets and actual and/or near-bankruptcies of some of the biggest financial institutions such as Lehman Brothers and AIG and the ensuing global recession, policymakers around the world responded by implementing fiscal stimulus packages consisting of tax breaks and substantial amounts of government spending, as can be seen from table 1, which shows the size of the fiscal stimulus package in a given year. The ensuing debate, both in academia and among policymakers, centers around the question of the effectiveness of fiscal policy: faced by a large fallout of demand, should the government step in and make up for the lost demand? We analyse this question for a particular set of circumstances that has become of relevance after the credit crisis, particularly in (Southern) Europe: an environment characterised by financial fragility, weakly capitalised banks and sovereign debt discounts in the face of widening public sector deficits. We argue that the effectiveness of fiscal stimuli is much reduced in such circumstances to the point of possibly becoming counterproductive as bank balance sheets come under stress and sovereign debt discounts rise in the face of deficit financed fiscal stimuli.

% of GDP	2009	2010
United States	1.98	1.77
Euro Area	0.83	0.73
Japan	2.42	1.79
Emerging Asia	2.16	2.01
Other Countries	0.85	0.43

Table 1: Table displaying total fiscal stimulus packages as percentage of GDP in early crisis years. *Source:* Coenen et al. (2010).

The focus of this paper is the negative amplification cycle that may arise from the feedback loops between weak banks and weak governments, and more specifically, the way in which the positive effects arising from additional demand from increased government purchases, are possibly reduced or even reversed, due to direct crowding out in what we call the credit provision channel of monetary transmission, and the capital losses on government bonds because of increased debt issuance. Due to this feedback mechanism, the room for governments to engage in additional policy is severely limited, a conclusion that contrasts with results from many contemporaneous macromodels, which assume the government has pockets deep enough to finance any possible intervention.

## *Empirical observations regarding the connection between weak banks and weak sovereigns*

For several reasons, the poisonous interaction between weak banks and weak governments is more of an issue in current day Europe than in the U.S. First, banks matter much more both for the financing of corporate debt and for the financing of public debt in Europe than they do in the U.S. Whereas banks account for approximately 40% of debt financing for U.S. non-financial corporations before the sub-prime crisis, with more than 60% of corporate debt being raised through bond issuance in capital markets, this ratio is more than reversed in the euro area: bank loans account for more than 80% of debt finance to non-financial corporations. This implies that non-financial corporates have more trouble obtaining debt funding when the banking sector is weakly

capitalised, whereas they have better access to capital markets in the U.S., and will therefore be less affected by financial fragility in the banking sector.

Besides this difference in financial architecture, the response to problems in the financial sector has been dramatically different in the U.S. and the euro area. The U.S. government announced its *Supervisory Capital Assessment Program* in March 2009 under which it stress-tested the big financial institutions, with mandatory recapitalisation in case of a capital shortfall. Losses were revealed and written down, and several banks raised substantial amounts of new capital through equity issuance in spite of distressed circumstances, most (97%) from private sources, as can be seen in figure 1. In Europe, on the other hand, stress-tests were not as severe as in the U.S., which left many financial institutions with hidden losses on their balance sheets (IMF 2011).

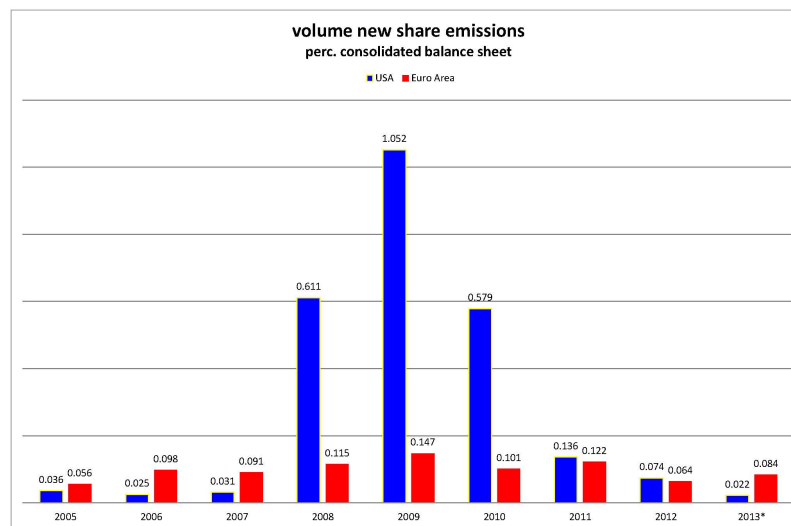


Figure 1: Figure displaying the volume of new share emissions of U.S. and Euro area banks. The volumes are denoted as percentage of the consolidated balance sheet of the banking system. \*Figure for 2013 is to 24-09-2013. *Source:* Thomson One Banker, BIS, ECB Consolidated Banking Data, as reported in DNB (2013).

The sovereign debt problems of several countries in the European periphery have added to the problems caused by financial fragility. Not only are European banks carrying unrecognized losses, they are also heavily exposed to domestic sovereign bonds, as can be seen from figure 2, which clearly shows the interconnectedness between sovereign default risk and the financial system. The figure presents the exposure of financial intermediaries to own country sovereign debt as a percentage of their Tier-1 capital. The exposure to the domestic sovereign is very substantial, which creates a strong link between the domestic sovereign and its banks. In all periphery countries except Cyprus, domestic sovereign debt exposure exceeds Tier-1 capital of the banks holding the debt, often by a substantial margin: in Spain banks' sovereign debt holdings equal 150% of Tier-1 capital, in Italy

almost 200% and in Greece almost 250%. Among the euro-core countries, Germany is a surprising outlier, with banks holding almost three times their Tier-1 capital in German sovereign debt. With domestic sovereign debt exposure so high among especially the Southern European banks, stress in the sovereign debt market can potentially have a very destabilizing impact on the financial system.

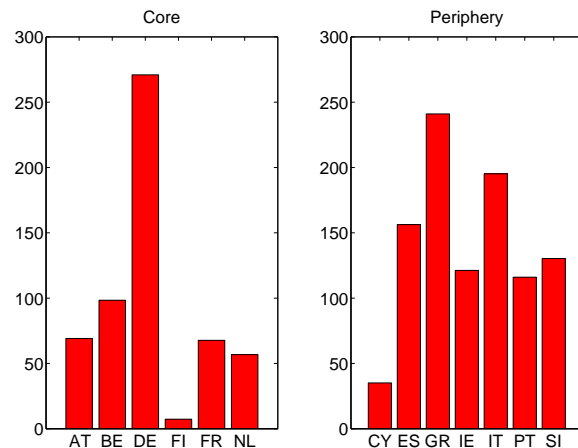


Figure 2: Figure displaying the exposure of banks to domestic sovereign debt (all maturities) as a percentage of their total Tier-1 capital in the core, respectively the periphery of the eurozone. *Source:* European Banking Authority (2011) and Keith Kuester, personal communication.

Figure 3 suggests that crowding out of private sector loans by public sector debt holdings is not just a theoretical artefact. This table shows the change in aggregate risk exposure<sup>2</sup> (EAD for Exposure At Default) for 16 European global systemically important banks (G-SIBs). The exposure to sovereign debt of the 16 major European banks considered has increased by EUR 550 billion, a 26% increase in sovereign EAD between end-of-2010 and end-of-2012. Over the same period, bank risk exposure to corporates has fallen by EUR 440 billion.

#### *Key features of the model*

To analyse the effectiveness of fiscal stimulus policies in the context of financial fragility, we construct a DSGE model with financial frictions building on Gertler and Karadi (2011), but extend it in several ways to accommodate the problems we want to highlight, in a way similar to Van der Kwaak and Van Wijnbergen (2014). In particular we allow banks to allocate their funds over corporate loans and long term government debt instruments subject to sovereign risk. In this way we combine an economy with financial intermediaries that are balance sheet constrained while holding long term government bonds subject to sovereign default risk. Through this channel we capture the interconnectedness between the financial system and (potential) fiscal/debt problems of the government. Long term government bonds are introduced in a way similar to Woodford (1998, 2001), with a variable maturity structure captured by a single parameter  $\rho$ , through which any

<sup>2</sup>Aggregate risk exposure refers to the part of the asset that exposes risk to the financial intermediary. Fitch Ratings (2013) reports for example, that the 16 G-SIBs represent end-2012 EUR 21 trillion in assets, which translates into to an aggregate risk exposure (EAD) of EUR 13.5 trillion.

## Risk Exposure (EAD): Modest Reduction, Major Reallocation

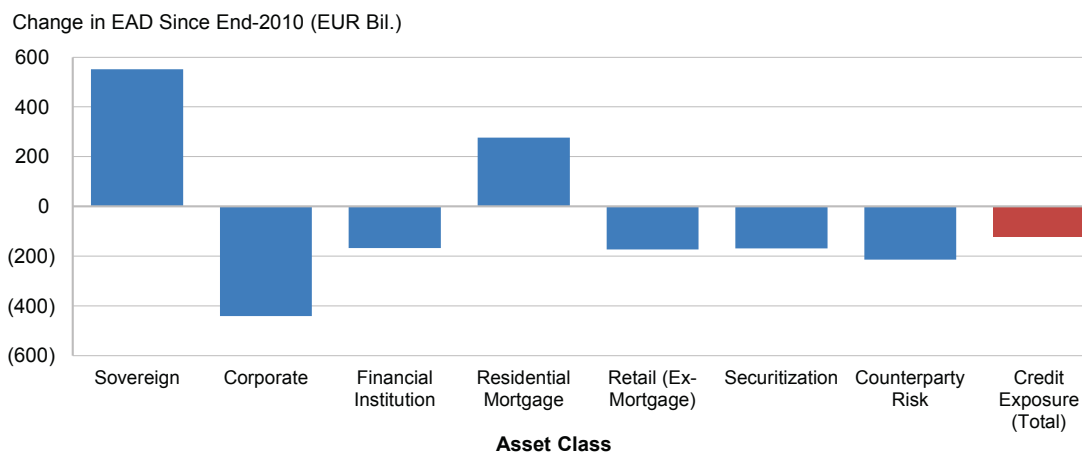


Figure 3: Bank pillar 3 disclosures from a sample of 16 European global systemically important banks (G-SIBs). EAD denotes Aggregate Exposure at Default, a measure for aggregate risk exposure. *Source: Fitch Ratings (2013).*

duration between 1 period bonds ( $\rho = 0$ ) and perpetuals, or ‘consols’ ( $\rho = 1$ ) can be obtained. The average maturity of domestic sovereign debt held by Southern European banks is approximately 5 years. Introducing longer maturities is important because of the link with capital losses for the already balance sheet constrained commercial banks in the model: the longer the maturity of the government bonds, the higher the capital losses for the financial intermediaries, and the more pronounced the adverse effects on the economy in case of a financial crisis. We should emphasize that we do not try to derive an optimal maturity structure, but, more modestly, show that exogenously lengthening the maturity structure strengthens the poisonous link between financial fragility and sovereign weakness in the debt market.

Sovereign default risk is captured by postulating a so called maximum level of taxation that is politically feasible. We then map this maximum level of taxation into a maximum level of debt. We assume that the government follows a core tax policy that aims for maintaining intertemporal solvency, but shocks to the system may necessitate issuing debt beyond this maximum level, in which case partial default follows to maintain debt below its maximum limit.

We use the model to explain the damaging amplification mechanism that can arise through interactions between capital losses on sovereign bonds, increasing sovereign risk, and crowding out mechanisms within the balance sheets of leverage constrained financial intermediaries when governments resort to deficit financed Keynesian demand management.

### *Relation to the literature*

Empirical evidence on the effectiveness of fiscal stimulus is mixed. Barro and Redlick (2011) find a multiplier of 0.7, which increases to unity when they allow for interactions with the unem-

ployment rate. Auerbach and Gorodnichenko (2011, 2012) and Bachmann and Sims (2012) show that the multiplier heavily depends on the state of the economy: it is moderate, or even negative in expansions, while it is larger than 2 in recessions. Blanchard and Perotti (2002), using the SVAR (Structural Vector Autoregression) approach, find a multiplier of 1 in the U.S. for government purchases. Corsetti et al. (2011) show that the multiplier is lower when debt is high (60% of GDP), but larger during financial crises. Ilzetzki et al. (2012), on the other hand, find that for countries with debt levels exceeding 60% of GDP, the impact multiplier is close to zero, and the long run multiplier -3, suggesting that debt sustainability is an important determinant for the output effect of fiscal stimulus. Theoretically results have been mixed as well: standard flexible-price neoclassical models always have multipliers smaller than unity, while new Keynesian models usually have a larger multiplier than their neoclassical equivalents, but dependent on the stance of monetary policy, and most of the time below unity as well, although Christiano et al. (2011) also investigate fiscal stimulus in a relatively standard New-Keynesian model and find that fiscal multipliers are significantly above 1 when the zero lower bound (ZLB) binds. Fernandez-Villaverde (2010) investigates the effects of fiscal policy in a model with financial frictions in the spirit of Bernanke et al. (1999), and finds that increased government spending is a more powerful tool to increase output in the short term than cutting distortionary taxes. Our model goes beyond this literature by looking specifically at debt financed fiscal stimuli in an environment with weakly capitalised banks and doubtful government finances (sovereign debt discounts), highlighting the negative feedbacks between these two features when banks are loaded up on government debt.

Since the start of the credit crisis, the theoretical literature with general equilibrium models containing financial frictions is booming. Possibly the first paper to incorporate financial frictions in a general equilibrium setup is Bernanke et al. (1999). We build on Gertler and Karadi (2011) who introduce financial intermediaries that are balance sheet constrained by an agency problem between the deposit holders and the bank owners. This gives rise to an endogenous leverage constraint, which becomes more binding when net worth is reduced by for example a negative shock to the quality of the loans. Several others have a similar mechanism, for example Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Kirchner and Van Wijnbergen (2012), who include financial intermediaries holding short term government debt besides loans to the private sector. The current paper extends the Kirchner-Van Wijnbergen (2012) model by introducing long term government bonds and sovereign default risk. Woodford (1998, 2001) introduces long term government debt by assuming that the government is financed through a bond with infinite maturity but declining coupons, thereby shortening the effective maturity. We follow his approach to modeling maturity. These papers, however, do not take into account the possibility of a government default. Acharya et al. (2011) have a setup containing both financial sector bailouts and sovereign default risk, but their analysis occurs within a partial equilibrium setup. Acharya and Steffen (2012), in their empirical research on systemic risk of the European banking sector, find that European banks have been at the center of the two major systemic crises that have faced the financial system since 2007, and specifically that markets have demanded more capital from banks with high sovereign debt exposures to peripheral countries, thereby indicating that sovereign debt holdings from those countries are a major contributor to systemic risk. A full general equilibrium model with banks financing government bonds subject to sovereign default risk is investigated by Kollmann et al. (2013). They have 1 period government bonds, and exogenous sovereign default risk, but no feedback from government debt to default risk.

Questions concerning the costs and benefits of long term government debt and the optimal maturity structure are discussed in Cole and Kehoe (2000), Chatterjee and Eyigungor (2012), and



Arellano and Ramanarayanan (2012). Of course, the optimal maturity structure depends on more than simply the risk and size of capital losses that financial intermediaries incur on their sovereign debt portfolio. But designing the optimal maturity structure is not the ambition of this paper, so we take maturity structure as exogenous. Staggered price setting and price stickiness go back to Calvo (1983) and Yun (1996). Sovereign default risk is captured in Arellano (2008), which contains an endogenous “strategic” default mechanism somewhat similar in outcome to our approach. She finds a maximum level of debt conditional on the income shock. Uribe (2006) contains a government that defaults on part of its government debt each period in such a way that the expected future tax receipts and liabilities match afterwards. Schabert and Van Wijnbergen (2011) introduce sovereign default risk by assuming that there exists a maximum level of taxation that is politically feasible and stochastic. Investors do not know this maximum level ex-ante, but they do know the distribution, and can deduce that the risk of a default is increasing with government debt. Davig et alii (2011) do not model an explicit default, but they do assume a maximum level of taxation, or ‘fiscal limit’, exists. Our approach is most related to the latter three contributions, and follows Van der Kwaak and Van Wijnbergen (2014).

## 2 Model description

The financial sector is based on endogenous leverage constraints like Gertler and Karadi (2011). But in our set up banks do not only lend to corporates but also to governments and we introduce long term government debt and endogenous sovereign default risk. The government issues debt to financial intermediaries and raises taxes in a lump sum fashion from the households to finance its expenditures and repay existing debt. Like in Gertler and Karadi (2011), the central bank sets the nominal interest rate on the deposits that the households bring to the financial intermediaries. The private sector consists of financial intermediaries and a non-financial sector (households and firms). The structure of the non financial private sector is relatively standard: capital producing firms buy investment goods and used capital, and convert these into new capital that is sold to the intermediate goods producers. The intermediate goods producers use the capital together with labor to produce intermediate goods for the retail firms. There is perfect competition in the intermediate goods market, and hence the ex-ante profits of the intermediate goods producers are zero in equilibrium. The retail firm repackages and sells his unique retail product to the final good producers, while exploiting his (local) monopoly power to charge a mark-up for his product. The final good producers buy these goods and combine them into a single output good. The final good is purchased by the households for consumption, by the capital producers to convert it into capital, and by the government. The household maximizes life-time utility subject to a budget constraint, which contains income from deposits, profits from the firms, both financial and non-financial, and from labor. The income is used for consumption, lump sum taxes and investments in deposits. The government can intervene and provide the financial sector with new capital (net worth). We describe the model below, but leave some of the more elaborate derivations for the appendix.

### 2.1 The household sector

There is a continuum of infinitely lived households with identical preferences and asset endowments. A typical household consists of bankers and workers. Every period, a fraction  $f$  of the household members is a banker running a financial intermediary. A fraction  $1 - f$  of the household members is a worker. At the end of every period, all members of the household pool their resources, and

every member of the household has the same consumption pattern. Hence there is perfect insurance within the household, and the representative agent representation is preserved. Every period, the household earns income from the labor of the working members and the profits of the firms that are owned by the household. In addition, households keep short term deposits in commercial banks, which are paid back with interest. The household uses these incoming cashflows to buy consumption goods which are consumed immediately upon purchase, and make new deposits into financial intermediaries.<sup>3</sup> The household members derive utility from consumption and leisure, with habit formation in consumption, in order to capture realistic consumption dynamics (Christiano et al. (2005)). Households optimize expected discounted utility:

$$\max_{\{c_{t+s}, h_{t+s}, d_{t+s}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \log(c_{t+s} - v c_{t-1+s}) - \Psi \frac{h_{t+s}^{1+\varphi}}{1+\varphi} \right) \right], \quad \beta \in (0, 1), \quad v \in [0, 1), \quad \varphi \geq 0$$

where  $c_t$  is consumption per household, and  $h_t$  are hours worked by the members of the household that are workers. The utility function is maximized subject to the following budget constraint:

$$c_t + d_t + \tau_t = w_t h_t + (1 + r_t^d) d_{t-1} + \Pi_t$$

Deposits  $d_{t-1}$  are posted at the financial intermediary in period  $t-1$ , and pay real interest  $r_t^d$  and principal at time  $t$ .  $w_t$  is the real wage rate,  $\tau_t$  are lump sum taxes the household has to pay to the government, and  $\Pi_t$  are the profits from the firms owned by the households. The profits of the financial intermediary are net of the startup capital for new bankers, as will be explained below. The first order conditions are then given by:

$$c_t : \lambda_t = (c_t - v c_{t-1})^{-1} - v \beta E_t [(c_{t+1} - v c_t)^{-1}] \quad (1)$$

$$h_t : \Psi h_t^\varphi = \lambda_t w_t \quad (2)$$

$$d_t : 1 = \beta E_t [\Lambda_{t,t+1} (1 + r_{t+1}^d)] \quad (3)$$

where  $\lambda_t$  is the Lagrange multiplier of the budget constraint, and the stochastic discount factor  $\Lambda_{t,t+i} = \lambda_{t+i}/\lambda_t$  for  $i \geq 0$ . The budget constraint is binding when the multiplier is larger (than zero ( $\lambda_t > 0$ )).

## 2.2 Financial intermediaries

The financial sector setup is similar to Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), except that we allow the financial intermediary to invest both in private loans and government bonds. Thus financial intermediaries lend funds obtained from households to the intermediate goods producers and to the government. We assume that households do not deposit with the bankers belonging to the same household, in order to prevent self-financing. Since we have a continuum of households with mass one, there are infinitely many other banks to which the household can lend its funds. The banker's balance sheet is given by:

$$p_{j,t} = n_{j,t} + d_{j,t}$$

where  $p_{j,t}$  are the assets on the balance sheet of bank  $j$  in period  $t$ ,  $n_{j,t}$  denotes the net worth of the bank, while  $d_{j,t}$  denotes the deposits of the bank. The financial intermediary invests the

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<sup>3</sup>but not in the ones owned by the family, in order to prevent self-financing.

funds obtained from the household in claims issued by the intermediate goods producer and in government bonds. Hence the asset side of the bank's balance sheet has the following structure:

$$p_{j,t} = q_t^k s_{j,t}^k + q_t^b s_{j,t}^b$$

where  $s_{j,t}^k$  are the number of claims the financial intermediary  $j$  has acquired for a price  $q_t^k$  from the intermediate goods producers, paying a net real return  $r_{t+1}^k$  at the beginning of period  $t + 1$ . The number of government bonds  $s_{j,t}^b$  are acquired by intermediary  $j$ , for a price  $q_t^b$ . At the beginning of period  $t + 1$  a net real return  $r_{t+1}^b$  is paid out. Financial intermediaries earn a return on their assets, and pay a return on the deposits. The difference between the two adds to the increase in net worth from one period to the next, possibly supplemented by government support measures. The balance sheet of intermediary  $j$  then evolves as follows:

$$\begin{aligned} n_{j,t+1} &= (1 + r_{t+1}^k)q_t^k s_{j,t}^k + (1 + r_{t+1}^b)q_t^b s_{j,t}^b - (1 + r_{t+1}^d)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (r_{t+1}^k - r_{t+1}^d)q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d)q_t^b s_{j,t}^b + (1 + r_{t+1}^d)n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t} \end{aligned}$$

where  $n_{j,t+1}^g = \tau_{t+1}^n n_{j,t}$  denotes net worth provided by the government to financial intermediary  $j$  (for example a capital injection).  $\tilde{n}_{j,t+1}^g = \tilde{\tau}_{t+1}^n n_{j,t}$  denotes the repayment of government support received in previous periods.

The financial intermediary is interested in maximizing expected profits. There is a probability of  $1 - \theta$  that the banker has to exit the industry next period, in which case he will bring the net worth  $n_{j,t+1}$  to the household, while he is allowed to continue operating with a probability  $\theta$ . The banker discounts these outcomes by the household's stochastic discount factor  $\beta\Lambda_{t,t+1}$ , since the banker is part of the household, the ultimate owner of the financial intermediary. The banker's objective is then given by the following recursive optimisation problem:

$$V_{j,t} = \max E_t \left[ \beta\Lambda_{t,t+1} \{ (1 - \theta)n_{j,t+1} + \theta V_{j,t+1} \} \right]$$

where  $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$ . We conjecture the solution to be of the following form, and later check whether this is the case:

$$V_{j,t} = \nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t}$$

We follow Gertler and Karadi (2011) and proxy governance problems in banks by assuming that bankers running the financial intermediaries can steal a certain fraction of the assets. If that happens, depositors will force the intermediary into bankruptcy, but in that case they can only recoup a fraction  $1 - \lambda$  of the assets stolen by the banker. Depositors, therefore, will in equilibrium only provide deposits up until the level where the continuation value of the intermediaries is equal to the value of the assets that can be diverted by the bankers (i.e. net of the part recovered by the depositors), so the bankers will actually choose to continue as banker. This imposes an endogenous leverage constraint on the financial intermediaries, lenders will only supply funds if the gains from stealing are lower than the continuation value of the financial intermediary. This gives rise to the following constraint:

$$V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) \tag{4}$$

The optimisation problem can now be formulated in the following way:

$$\max_{\{q_t^k s_{j,t}^k, q_t^b s_{j,t}^b\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b)$$

From the first order conditions we find that  $\nu_t^b = \nu_t^k$ . Hence the leverage constraint (4) can be rewritten in the following way:

$$q_t^k s_{j,t}^k + q_t^b s_{j,t}^b \leq \phi_t n_{j,t}, \quad \phi_t = \frac{\eta_t}{\lambda - \nu_t^k} \quad (5)$$

where  $\phi_t$  denotes the ratio of assets to net worth, which can be seen as the leverage constraint of the financial intermediary. The intuition for the leverage constraint is straightforward: a higher shadow value of assets  $\nu_t^k$  indicates a higher value from an additional unit of assets, increasing expected profits everything else equal, thereby reducing the incentive for bankers to steal the assets. A higher value of  $\eta_t$  implies higher expected profits from an additional unit of net worth, therefore allowing a higher leverage ratio. A higher fraction  $\lambda$  implies bankers can steal more, inducing households to provide less deposits everything else equal. After substitution of the conjectured solution into the right hand side of the Bellman equation, application of the first order conditions, and comparing the left and right hand side of the Bellman equation, we obtain the following expressions for the shadow values of net worth, private loans, and government bonds:

$$\eta_t = E_t \left[ \Omega_{t+1} (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n) \right] \quad (6)$$

$$\nu_t^k = E_t \left[ \Omega_{t+1} (r_{t+1}^k - r_{t+1}^d) \right] \quad (7)$$

$$\nu_t^b = \nu_t^k = E_t \left[ \Omega_{t+1} (r_{t+1}^b - r_{t+1}^d) \right] \quad (8)$$

$$\Omega_{t+1} = \beta \Lambda_{t,t+1} \{ (1 - \theta) + \theta [\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}] \}$$

where  $\Omega_{t+1}$  can be thought of as a stochastic discount factor that incorporates the financial friction. A detailed derivation is provided in the appendix.

### 2.2.1 Aggregation of financial variables

Aggregating the balance sheet identities is straightforward. Since  $\phi_t$  does not depend on firm specific factors, we can aggregate the leverage constraint (5) across financial intermediaries to find the total number of assets:

$$p_t = q_t^k s_t^k + q_t^b s_t^b = \phi_t n_t \quad (9)$$

where  $p_t$  denotes the aggregate quantity of assets that are on the balance sheets of the financial intermediaries, while  $n_t$  denotes the aggregate intermediary net worth. The share of assets invested in private loans is given by:

$$\omega_t = q_t^k s_t^k / p_t \quad (10)$$

At the same time, we know that at the end of the period, only a fraction  $\theta$  of the current bankers will remain a banker, while the remaining fraction  $1 - \theta$  will (again) become a worker. We assume that current bankers only pay out dividends at the moment they quit the banking business. If they do not quit but continue as a banker, they retain their earnings to expand their net worth and expand their balance sheet. Thus the net worth of existing bankers at the end of the period is equal to:

$$n_{e,t} = \theta [(r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1}]$$

The exiting bankers bring back the net worth to the household income. A fraction  $1 - \theta$  of the bankers has left the financial industry, which is equal to a fraction  $(1 - \theta)f$  of the households. The same fraction of the households will enter the financial industry next period by leaving their working job. We assume that the household will provide a starting net worth to the new bankers proportional to the assets of the old bankers, as in Gertler and Karadi (2011). We assume that the household transfers a fraction of  $\chi/(1 - \theta)$  of the assets of the old bankers to the new bankers. Then aggregate net worth of new bankers equals:

$$n_{n,t} = \chi p_{t-1}$$

At the beginning of the new period, random draws determine which bankers will leave the industry, taking their accumulated net worth with them. Total net worth of the bankers active in the new period (old bankers that continue plus new bankers and possibly government support) equals:

$$\begin{aligned} n_t &= n_{e,t} + n_{n,t} + n_t^g - \tilde{n}_t^g \\ &= \theta [(r_t^k - r_t^d)q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d)q_{t-1}^b s_{t-1}^b + (1 + r_t^d)n_{t-1}] \\ &+ \chi p_{t-1} + n_t^g - \tilde{n}_t^g \end{aligned} \tag{11}$$

where  $n_t^g$  and  $\tilde{n}_t^g$  are aggregate financial sector support, respectively financial support payback.

## 2.3 Production side

There is a continuum of intermediate goods producers indexed by  $i \in [0, 1]$  that face perfect competition. They borrow funds from the financial intermediaries to purchase the capital necessary for production. With the proceeds from the sale of the output and the sale of the capital after it has been used, the firms will pay workers and pay back the loans to the financial intermediary. The output is sold to the retail firms. The capital producers buy and “recycle” the used (and partially depreciated) old capital; they also produce new capital (investment) from goods purchased from the final goods producers. The recycled old capital and the new investments combine to form the new capital stock at the beginning of the next period. This new capital is sold to the intermediate goods producers, which will use it for production next period. A continuum of retail firms, indexed by  $f \in [0, 1]$ , repackage the products bought from the intermediate goods producers to produce a unique differentiated retail product. The retail firms sell their products to a continuum of final goods producers. Due to the fact that the products are differentiated, each individual retail firm faces a monopolistic competitive market, and can charge a markup. It can only change prices in a period, though, with a certain probability, which is equal to  $\psi$  and i.i.d. for every retail firm. Hence some firms are allowed to reset prices while others are not. The final goods producers have a technology to convert the inputs from the retail firms into final goods. Due to perfect competition, the profits will be zero in equilibrium, and the final goods are sold to the households, the government, and the capital producers, which use it as an investment to produce new capital for use in the next period by the intermediate goods producing firms.

### 2.3.1 Capital Producers

In this section we describe the capital producers. At the end of period  $t$ , when the intermediate goods firms have produced, they sell what remains of the capital stock after depreciation  $\delta$  to the capital producers at a price  $q_t^k$ . The capital producers also buy  $i_t$  final goods from the final good

producers; these purchases (investment) are an input in the capital production process: they are used to produce additional capital. Capital producers combine this additional capital with the old, partially depreciated stock bought earlier from the intermediate goods producers and so produce the new capital stock. This new capital is being sold to the intermediate goods producers at a price  $q_t^k$ . We assume that the capital producers face convex adjustment costs whenever investment  $i_t$  deviates from previous period investment  $i_{t-1}$ . These adjustment costs are the reason that one unit of investment goods cannot be transformed into one unit of capital, unless  $i_t = i_{t-1}$ . Hence we have the following capital production technology:

$$k_t = (1 - \delta)\xi_t k_{t-1} + (1 - \Psi(\iota_t))i_t, \quad \Psi(x) = \frac{\gamma}{2}(x - 1)^2, \quad \iota_t = i_t/i_{t-1} \quad (12)$$

The capital producers are profit maximizing, and profits are passed on to the households, who are the owners of the capital producers. The profit in period  $t$  is given by:

$$\Pi_t^c = q_t^k k_t - q_t^k (1 - \delta)\xi_t k_{t-1} - i_t$$

The capital producers' optimization problem is then given by:

$$\max_{\{i_{t+i}\}_{i=0}^{\infty}} E_t \left[ \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left( q_{t+i}^k (1 - \Psi(\iota_{t+i})) i_{t+i} - i_{t+i} \right) \right]$$

Differentiation with respect to investment gives the first order condition for the capital producers:

$$q_t^k (1 - \Psi(\iota_t)) - 1 - q_t^k \iota_t \Psi'(\iota_t) + \beta E_t \Lambda_{t,t+1} q_{t+1}^k \iota_{t+1} \Psi'(\iota_{t+1}) = 0$$

This equation can be rewritten to find the price of capital to be:

$$\frac{1}{q_t^k} = 1 - \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \frac{\gamma i_t}{i_{t-1}} \left( \frac{i_t}{i_{t-1}} - 1 \right) + \beta E_t \left[ \Lambda_{t,t+1} \frac{q_{t+1}^k}{q_t^k} \left( \frac{i_{t+1}}{i_t} \right)^2 \gamma \left( \frac{i_{t+1}}{i_t} - 1 \right) \right] \quad (13)$$

### 2.3.2 Intermediate Goods Producers

There exists a continuum of intermediate goods producers indexed by  $i \in [0, 1]$ . Each of these firms produce a differentiated good. The intermediate goods producers borrow from the financial intermediaries against future profits. We assume that there are no financial frictions between the financial intermediaries and the intermediate goods producers. Hence there are no monitoring costs for the financial intermediaries, and the intermediate goods producers can commit next period's profits to the financial intermediaries. The securities issued by the intermediate goods producers are therefore really state-contingent debt, see also Gertler and Kiyotaki (2010)<sup>4</sup>. The production technology of the intermediate goods producers is given by:

$$\begin{aligned} y_{i,t} &= a_t (\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha}, \\ \log(a_t) &= \rho_a \log(a_{t-1}) + \varepsilon_{a,t} \\ \log(\xi_t) &= \rho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t} \end{aligned}$$

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<sup>4</sup>The claims of financial intermediaries can therefore be better thought of as equity. Occhino and Pescatori (2010) explicitly model loans to producers with a fixed face value, and include the possibility of a default by the goods producers. We refrain from explicitly modelling the producers' default, and note the equity characteristics of debt in the real world when firms have not enough funds to pay off the loan.

where  $a_t$  equals total factor productivity and  $\xi_t$  capital quality. The innovations  $\varepsilon_{x,t}$  are distributed as  $\varepsilon_{x,t} \sim N(0, \sigma_x^2)$  for  $x = a, \xi$ . The intermediate goods producer acquires the capital at the end of period  $t - 1$ , while the production only occurs after the capital quality shock  $\xi_t$  has hit at the beginning of period  $t$ . Hence  $\xi_t k_{i,t-1}$  denotes the effective capital in our model. We see that if a negative realization of  $\varepsilon_{\xi,t}$  occurs, the quality of the capital deteriorates immediately. Hence the firm will not be able to produce as much as when the shock does not occur. Remember that the number of claims ( $s_{i,t}^k$ ) is equal to the number of units of capital purchased ( $k_{i,t}$ ); hence the return on the claims of the financial intermediary will be lower. We can think of the shock  $\xi_t$  as accelerated economic depreciation or obsolescence of capital. The intermediate goods producer decides at the end of period  $t - 1$  how much capital to purchase. At the moment the intermediate goods producer purchases the capital, he does not know the realization of  $\xi_t$  in period  $t$  yet. To finance his purchase at the end of period  $t - 1$ , he needs to issue claims  $s_{i,t-1}^k$ , with the number of claims  $s_{i,t-1}^k$  equal to the number of capital units ( $k_{i,t-1}$ ) acquired. The price at which the claims are sold equals  $q_{t-1}^k$ , and they pay a state-contingent net real return  $r_t^k$  in period  $t$ . The intermediate goods producer also hires labor  $h_{i,t}$  for a wage rate  $w_t$  after the shock ( $\xi_t$ ) has been realized. When the firm has produced in period  $t$ , the output is sold for a relative price  $m_t$  to the retail firms.  $m_t$  is the relative price of the intermediate goods with respect to the price level of the final goods, i.e.  $m_t = P_t^m / P_t$ . After production, the intermediate goods producing firms sell back what remains of the effective capital to the capital producers at a price of  $q_t^k$ . The effective capital stock is also subject to regular depreciation  $\delta$  during production. So the intermediate goods producer receives  $q_t^k(1 - \delta)\xi_t k_{i,t-1}$  for his end of period capital stock and real profits in period  $t$  are given by:

$$\Pi_{i,t} = m_t a_t (\xi_t k_{i,t-1})^\alpha h_{i,t}^{1-\alpha} + q_t^k (1 - \delta) \xi_t k_{i,t-1} - (1 + r_t^k) q_{t-1}^k k_{i,t-1} - w_t h_{i,t}$$

The objective of the intermediate goods producing firms is to maximize gross profits. They take the relative output price ( $m_t$ ), and the input prices  $q_t^k, r_t^k$  and  $w_t$  as given when maximizing profits. The first order condition with respect to capital and labor are given by:

$$\begin{aligned} k_{i,t} &: E_t \left[ \beta \Lambda_{t,t+1} q_t^k (1 + r_{t+1}^k) \right] = E_t \left[ \beta \Lambda_{t,t+1} (\alpha m_{t+1} y_{i,t+1} / k_{i,t} + q_{t+1}^k (1 - \delta) \xi_{t+1}) \right] \\ h_{i,t} &: w_t = (1 - \alpha) m_t y_{i,t} / h_{i,t} \end{aligned}$$

Firms pay out residual revenues to the financial intermediaries. By substituting the first order condition for the wage rate into the zero-profit condition  $\Pi_{i,t} = 0$ , we can find an expression for the ex-post return on capital:

$$r_t^k = (q_{t-1}^k)^{-1} (\alpha m_t y_{i,t} / k_{i,t-1} + q_t^k (1 - \delta) \xi_t) - 1$$

The first order condition for labor and the expression for the ex-post return on capital can be rearranged to derive factor demands. These are given by:

$$\begin{aligned} k_{i,t-1} &= \alpha m_t y_{i,t} / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta) \xi_t] \\ h_{i,t} &= (1 - \alpha) m_t y_{i,t} / w_t \end{aligned}$$

Finally the relative intermediate output price  $m_t$  can be obtained by substituting the factor demands into the production function:

$$m_t = \alpha^{-\alpha} (1 - \alpha)^{\alpha-1} a_t^{-1} \left( w_t^{1-\alpha} [q_{t-1}^k (1 + r_t^k) \xi_t^{-1} - q_t^k (1 - \delta)]^\alpha \right) \quad (14)$$

### 2.3.3 Retail firms

Retail firms purchase goods ( $y_{i,t}$ ) from the intermediate goods producing firms for a nominal price  $P_t^m$ , and convert these into retail goods ( $y_{f,t}$ ). These goods are sold for a nominal price  $P_{f,t}$  to the final goods producer. We assume that it takes one intermediate goods unit to produce one retail good ( $y_{i,t} = y_{f,t}$ ). All the retail firms produce a differentiated retail good. The individual retail firm therefore operates in a market characterized by monopolistic competition, and can therefore charge a markup over the input price  $P_t^m$ , so the nominal profit of the retail firm is given by  $(P_{f,t} - P_t^m)y_{f,t}$ .

We assume that in each period a fraction  $1 - \psi$  of retail firms is allowed to reset their prices optimally, while the  $\psi$  remaining firms are not allowed to do so. This is set up like in Calvo (1983) and Yun (1996), so we assume that this probability is independent of the probability that other producers are allowed to reset their prices, and independent of whether or not the firm was allowed to adjust its prices the previous period. Hence the probability that a firm is not allowed to reset its prices is  $\psi$ , where  $\psi$  is i.i.d. When a firm is allowed to reset its prices, it will do it in such a way that the expected sum of discounted profits is maximized. Again, the retail firms are owned by the households, and therefore the stochastic discount factor for the nominal pay-outs to the household is given by  $\beta^s \Lambda_{t,t+s}(1/P_{t+s})$ . The relevant part of the optimization problem of the typical retail firm is now given by:

$$\max_{P_{f,t}} E_t \left[ \sum_{s=0}^{\infty} (\beta\psi)^s \Lambda_{t,t+s} \left[ \frac{P_{f,t}}{P_{t+s}} - m_{t+s} \right] y_{f,t+s} \right]$$

where  $y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} y_t$  is the demand function.  $y_t$  is the output of the final good producing firms, and  $P_t$  the general price level. The resulting first order conditions can be found in the appendix.

### 2.3.4 Final Good Producers

The final good firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by  $y_t^{(\epsilon-1)/\epsilon} = \int_0^1 y_{f,t}^{(\epsilon-1)/\epsilon} df$ , where  $y_{f,t}$  is the output of the retail firm indexed by  $f$ .  $\epsilon$  is the elasticity of substitution between the intermediate goods purchased from the different retail firms. We assume that the final good firms operate in an environment of perfect competition, and hence they maximize profits by choosing  $y_{f,t}$  such that  $P_t y_t - \int_0^1 P_{f,t} y_{f,t} df$  is maximized. The final good producer takes  $P_t$  and  $P_{f,t}$  as given. Taking the first order conditions with respect to  $y_{f,t}$  gives the demand function of the final good producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final good and the price level of the individual retail firms:

$$\begin{aligned} y_{f,t} &= (P_{f,t}/P_t)^{-\epsilon} y_t \\ P_t^{1-\epsilon} &= \int_0^1 P_{f,t}^{1-\epsilon} df \end{aligned}$$



### 2.3.5 Aggregation

Price dispersion, which is given by  $\mathcal{D}_t = \int_0^1 (P_{f,t}/P_t)^{-\epsilon} df$  is given by the following recursive form:

$$\mathcal{D}_t = (1 - \psi)(\pi_t^*)^{-\epsilon} + \psi\pi_t^\epsilon \mathcal{D}_{t-1} \quad (15)$$

Aggregation over the intermediate goods production technology gives the following relation for aggregate output:

$$y_t \mathcal{D}_t = a_t (\xi_t k_{t-1})^\alpha h_t^{1-\alpha} \quad (16)$$

## 2.4 The Government

The government levies lump sum taxes on households and issues bonds to finance its (exogenous) expenditures  $g_t$  and to pay back the principal of bonds that come due this period. We also allow the government to support the financial sector by providing additional net worth  $n_t^g$  when the quality of the capital, and hence the value of the financial sector's assets, deteriorates due to a negative capital quality shock. Repayment of previously administered support  $\tilde{n}_t^g$  by financial intermediaries provides the government with additional revenue. Government financing is modeled in a similar way as in Woodford (1998, 2001) in order to have a flexible maturity structure for the government bonds. Bonds  $b_t$  issued in period  $t$  provide the government with funds  $q_t^b b_t$  in real terms, where  $q_t^b$  is the real price of the outstanding bonds, and  $b_t$  the number of bonds. One government bond entitles the investor to a payment  $r_c$  fixed in real terms in period  $t+1$ ,  $\rho r_c$  real units in period  $t+2$ ,  $\rho^2 r_c$  real units in period  $t+3$  etc. This pay off structure is equivalent to entitling a bond holder to a full payment of  $r_c$  in the first period only, plus at the end of that period a new similar bond equal to  $\rho$  and so on. The case of one period bonds is captured by  $\rho = 0$ , while the case of perpetual bonds, or 'consols' is obtained by setting  $\rho = 1$ . Any value of  $\rho \in [0, 1]$  gives a maturity structure in between these 2 extremes. Government liabilities at the beginning of period  $t$  are therefore given by  $(r_c + \rho q_t^b) b_{t-1}$ . The average maturity<sup>5</sup> is  $1/(1 - \beta\rho)$ . This gives the following budget constraint in real terms:

$$\begin{aligned} q_t^b b_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (r_c + \rho q_t^b) b_{t-1} = g_t + n_t^g + \left( \frac{r_c + \rho q_t^b}{q_{t-1}^b} \right) q_{t-1}^b b_{t-1} \implies \\ q_t^b b_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (1 + r_t^b) q_{t-1}^b b_{t-1} \end{aligned} \quad (17)$$

where  $r_t^b$  is the real return on government bonds:

$$1 + r_t^b = \frac{r_c + \rho q_t^b}{q_{t-1}^b} \quad (18)$$

We assume that the government follows a simple fiscal rule for its core tax policy  $\tau_t$ , as in Bohn (1998):

$$\tau_t = \bar{\tau} + \kappa_b (b_{t-1} - \bar{b}) + \kappa_g (g_t - \bar{g}) + \kappa_n n_t^g, \quad \kappa_b \in (0, 1], \quad \kappa_g, \kappa_n \in [0, 1] \quad (19)$$

where  $\bar{b}$ ,  $\bar{g}$  are the steady state levels of debt, respectively government spending. The Bohn (1998) policy rule formulation guarantees that the real value of public debt eventually grows at a rate

<sup>5</sup>The duration (average maturity) equals:  $\frac{\sum_{j=1}^{\infty} j \beta^j (\rho^{j-1} r_c)}{\sum_{j=1}^{\infty} \beta^j (\rho^{j-1} r_c)} = 1/(1 - \beta\rho)$

smaller than the gross real rate of interest. Bohn (1998) proves that following this rule is a sufficient condition for government solvency. If we set  $\kappa_n = 0$ , the additional government transfers to the financial sector are completely financed by issuing new debt.  $\kappa_n = 1$  implies that the additional spending is completely financed by increasing lump sum taxes. Similarly  $\kappa_g = 0$  implies that all government spending above the steady state value is completely deficit financed, whereas  $\kappa_g = 1$  implies a tax-financed government spending stimulus.

Finally, consider the government's purchases of final goods in more detail. Government purchases are partially driven by an exogenous stochastic process, and in addition can possibly respond to a recession caused by an adverse quality of capital shock. The latter component would correspond to a Keynesian stimulus package, in that case the government attempts to stimulate the economy by increasing government purchases. Combining these two components yields the actual time path for government expenditures/purchases of final goods in period  $t$ :

$$g_t = \tilde{g}_t + \varsigma(\xi_{t-l} - \xi), \quad \varsigma < 0, \quad l \geq 0 \quad (20)$$

$$\log(\tilde{g}_t/\bar{g}) = \rho_g \log(\tilde{g}_{t-1}/\bar{g}) + \varepsilon_{g,t}^u + \varepsilon_{g,t-4}^a \quad (21)$$

where  $\varepsilon_g^x \sim N(0, \sigma_x^2)$  with  $x = (u, a)$ . We assume that the autocorrelation coefficient  $\rho_g \in [0, 1)$ , and the steady state value of government purchases to be larger than zero ( $\bar{g} > 0$ ). This way we can study the effects of surprise shocks to government spending ( $\varepsilon_{g,t}^u$ ), but also the effects of shocks that are announced one year in advance ( $\varepsilon_{g,t}^a$ ). The parameter  $\varsigma$  determines the size of the response to a deterioration in the quality of capital. If  $\varsigma = 0$ , the government does not respond to a deterioration in the quality of capital.  $\varsigma < 0$  implies that the government reacts to a deterioration in the quality of capital by increasing government spending above the steady state value. The case where  $l = 0$  implies that the government reacts instantaneously to a capital quality shock, while  $l > 0$  implies that the government reacts with some lag. Whereas it might be preferable in general to model the government response as an endogenous optimizing feedback from output, we choose to model government intervention as an exogenous process because of our focus on the size of fiscal multipliers. This allows us to make policy impact comparisons that are not "polluted" by second round effects triggered by the macroeconomic response to government expenditure shocks leading to subsequent rounds of government interventions.

## 2.5 The Central Bank

The Central Bank sets the nominal interest rate on deposits  $r_t^n$  according to the following Taylor rule, in order to minimize output and inflation deviations:

$$r_t^n = (1 - \rho_r)(r^n + \kappa_\pi(\pi_t - \bar{\pi}) + \kappa_y \log(y_t/y_{t-1})) + \rho_r r_{t-1}^n + \varepsilon_{r,t} \quad (22)$$

where  $\varepsilon_{r,t} \sim N(0, \sigma_r^2)$ , and  $\kappa_\pi > 0$  and  $\kappa_y > 0$ .  $\rho_r$  is a smoothing parameter. The parameter  $\bar{\pi}$  is the target inflation rate or the natural inflation rate. In order to satisfy the Taylor principle, we choose  $\kappa_\pi > 1$  (leaning against the wind). The values of  $\kappa_\pi$  and  $\kappa_y$  determine the strength with which the authorities react to deviations from the natural rate of inflation and output. The nominal and the real interest rate on deposits are linked through the following Fisher relation:

$$1 + r_t^d = (1 + r_{t-1}^n)/\pi_t \quad (23)$$

Hence monetary policy is executed through the control of interest rates on deposits rather than the interest rates on the government bonds: the latter are endogeneously determined in equilibrium.

## 2.6 Market clearing

Equilibrium requires that the number of claims owned by the financial intermediaries ( $s_t^k$ ) is equal to aggregate capital ( $k_t$ ), while the number of government bonds owned by the financial sector ( $s_t^b$ ) must equal the number of bonds issued by the government ( $b_t$ ):

$$s_t^k = k_t \quad (24)$$

$$s_t^b = b_t \quad (25)$$

Goods market clearing requires that the aggregate demand equals aggregate supply:

$$c_t + i_t + g_t = y_t \quad (26)$$

This completes the description of the model.

## 3 Extension with government default

In this section we will introduce the possibility of a (partial) government default. We assume that a (partial) default occurs because there is a maximum level of taxation that is politically feasible, similar to the fiscal limit as in Davig et alii (2011). We will show that such a maximum level of taxation corresponds to a maximum level of debt that can be issued. When the government has to issue more debt than this maximum level of debt, the government will partially default on its current liabilities. This process can be thought off as a debt restructuring, like in Greece 2012, in which creditors changed their government bonds for new ones with lower face value (or equal but less in number).

### Default process

The fact that governments can default changes the expectations of investors. When this is not the case, the only condition to be satisfied is the requirement that the debt grows at a smaller rate than the gross real rate of interest. In that case we have Ricardian equivalence, and an infinite number of government debt paths are possible. But when expectations of default influence debt prices, the path of government debt is not neutral anymore for the equilibrium allocation. Investors have rational expectations. We assume that there is a (fixed) maximum level of taxes that can be raised beyond which tax rates are not politically sustainable anymore, like in Schabert and Van Wijnbergen (2011), and similar to the ‘fiscal limit’, as in Davig et alii (2011). Call this maximum level  $\tau_t^{max}$ . This translates in a maximum sustainable level of debt  $b_t^{max}$  that can be sustained:

$$b_t^{max} = \bar{b} + \frac{\tau_{t+1}^{max} - \bar{\tau}}{\kappa_b} \quad (27)$$

At the beginning of period  $t$ , the government has to pay the coupon payment  $r_c$  on the bonds issued at the end of period  $t - 1$ . Besides that, the government has to buy back the old bonds for a price  $\rho q_t^b b_{t-1}$ , where  $q_t^b$  is the price at which the bonds in period  $t$  are being traded. The government liabilities  $L_t^q$  at the beginning of period  $t$ , before the fraction of debt over which the government defaults is determined, are given by:

$$L_t^q = (r_c + \rho q_t^b) b_{t-1}$$

where  $L_t^q$  is indicated by  $q$ , denoting the dependence of the government liabilities on the market valuation of government debt. The flow budget constraint of the government in period  $t$  in case of no government default is now given by the following expression:

$$q_t^b \tilde{b}_t = L_t^q + g_t + n_t^g - \tau_t - \tilde{n}_t^g$$

where  $\tilde{b}_t$  denotes the level of government debt if the government would not default on its obligations in the current period. The constraint can be rewritten in the following way:

$$q_t^b \tilde{b}_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + (r_c + \rho q_t^b) b_{t-1} \quad (28)$$

### Default rule

Now we come to the actual default rule that the government follows. As long as the level of debt that the government needs to issue in order not to default ( $\tilde{b}_t$ ) is smaller than the maximum level of debt  $b_t^{max}$ , the actual government debt  $b_t$  will be equal to the no default level of government debt  $\tilde{b}_t$ . When  $\tilde{b}_t > b_t^{max}$ , the government defaults on the fraction of debt such that the new debt  $b_t$  is equal to the maximum level of debt  $b_t^{max}$ :

$$b_t = \begin{cases} \tilde{b}_t & \text{if } \tilde{b}_t \leq b_t^{max}; \\ b_t^{max} & \text{if } \tilde{b}_t > b_t^{max}. \end{cases} \quad (29)$$

The government achieves this outcome through renegotiation with its creditors. Since creditors have rational expectations, they know that they will not be able to get more from the government than what the government can raise sustainably through the maximum level of taxes  $\tau^{max}$ . We assume all creditors participate in the renegotiation, and abstract from free-rider problems among creditors. We assume that they are willing to partially renege on the current payment  $r_c$ , and will only be paid out  $(1 - \Delta_t)r_c$  per bond issued in period  $t - 1$ , where  $\Delta_t$  denotes the fraction of debt over which the government defaults at the beginning of period  $t$ . Besides that they are willing to convert their old bonds into new ones at a rate of  $(1 - \Delta_t)b_{t-1}$ . We can see the debt structure  $b_t$  as the solid blue line in Figure 4. We can also write the debt level structure (29) in the following way:

$$b_t = \min(\tilde{b}_t, b_t^{max}) = b_t^{max} - \max(b_t^{max} - \tilde{b}_t, 0). \quad (30)$$

We can interpret the second term of the new debt level as the payoff of a put option at maturity with underlying process  $\tilde{b}_t$  and strike price  $b_t^{max}$ , similar to the debt pricing model presented in Claessens and Van Wijnbergen (1993). The debt-level, or default function, however, is not differentiable at  $b_t = b_t^{max}$ . In order to be able to solve the model using perturbation techniques, which require differentiability, we convexify (30) by replacing it by the corresponding put option valuation formula (see Figure 4). This is described in detail in appendix 9.1.

### 3.1 Government budget constraint and default

In case  $\tilde{b}_t \leq b_t^{max}$ , the government does not default, hence  $\Delta_t = 0$ , where  $\Delta_t$  denotes the fraction of debt over which the government defaults. In case the government does default, it scales back all current and future interest obligations associated with the debt stock carried into the default period. In the Woodford (1998, 2001) framework we use, this implies that the government saves

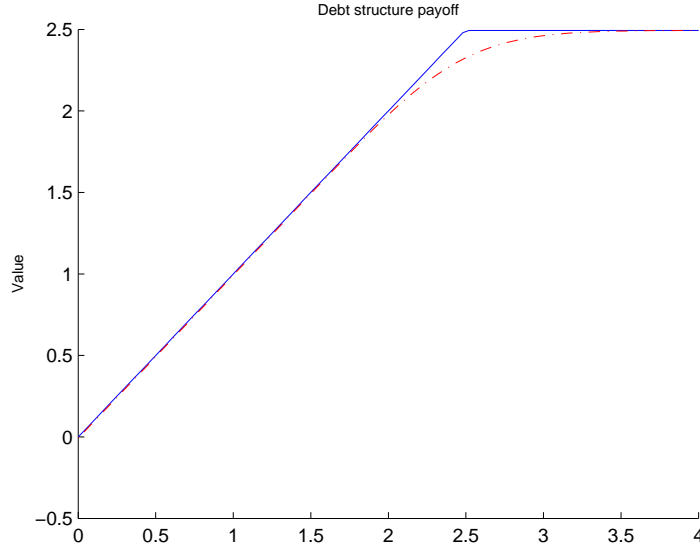


Figure 4: Plot showing the mapping from the no default level of debt  $\tilde{b}_t$  to the actual debt level  $b_t$ . The solid blue line is the actual mapping, while the slotted red line is an approximation to it, where the corresponding put option valuation formula has been used.

an amount equal to  $\Delta_t(r_c + \rho q_t^b)b_{t-1}$  on new debt issuance. The flow budget constraint of the government in period  $t$  in case of a government default is now given by the following expression:

$$q_t^b b_t = L_t^g + g_t + n_t^g - \tau_t - \tilde{n}_t^g - \Delta_t(r_c + \rho q_t^b)b_{t-1}$$

Hence the government budget constraint is given by:

$$q_t^b b_t + \tau_t + \tilde{n}_t^g = g_t + n_t^g + (1 - \Delta_t)(r_c + \rho q_t^b)b_{t-1} \quad (31)$$

### 3.2 Financial intermediaries and default

In this section we provide the equations of the financial intermediaries that are changed due to the introduction of a default. A more elaborate derivation can be found in the appendix. Similarly to the case of no default, we find that the law of motion of the net worth of an individual intermediary is given by:

$$n_{j,t+1} = (r_{t+1}^k - r_{t+1}^d) q_t^k s_{j,t}^k + (r_{t+1}^{b*} - r_{t+1}^d) q_t^b s_{j,t}^b + (1 + r_{t+1}^d) n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t}$$

where  $r_t^{b*}$  is given by:

$$1 + r_t^{b*} = (1 - \Delta_t) \left( 1 + r_t^b \right) = (1 - \Delta_t) \left( \frac{r_c + \rho q_t^b}{q_{t-1}^b} \right) \quad (32)$$

We note that the law of motion is exactly the same as for the case without government default, except for the replacement of  $r_t^b$  by  $r_t^{b*}$ . This implies that the expression for the leverage constraint

remains unchanged, as well as the expressions for the shadow value of private loans and net worth, and adjust the equations for the shadow value of government bonds, and the law of motion of net worth:

$$\nu_t^b = E_t \left[ \Omega_{t+1} (r_{t+1}^{b*} - r_{t+1}^d) \right] \quad (33)$$

$$n_t = \theta \left[ (r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^{b*} - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \right] + \chi p_{t-1} + n_t^g - \tilde{n}_t^g \quad (34)$$

The other equations for the financial intermediaries remain the same.

## 4 Calibration

Before using the model for simulation analysis, we need to assign numerical values to the parameters. For comparability with the existing literature, we choose values that are common in the literature on DSGE models or frequently used in models containing financial frictions such as Gertler-Karadi (2011). The calibration of the default process is explained in more detail in appendix 9.2. We calibrate the model on a quarterly frequency. The parameter values can be found in table 2. The steady state leverage ratio is set to 4, while the credit spread  $\Gamma$  is set to 100 basis points annually (which amounts to  $\Gamma = 0.0025$ ), which coincides with the pre-2007 spreads in US financial data between BAA corporate and government bonds. The parameter  $\theta$  is calibrated by taking the average survival period ( $\Theta = 1/(1 - \theta)$ ) to be equal to 36 quarters, or  $\theta = 0.9722$ . The parameters in the Taylor rule are set to conventional values.

The feedback from government debt on taxes is set to a value such that both the model with and without default are stable. In order to match US macroeconomic data, we calibrate the steady state ratios of investment and government spending over GDP,  $\bar{i}/\bar{y}$  and  $\bar{g}/\bar{y}$  to 20 percent by calibrating the depreciation parameter  $\delta$ . The fixed payment in real terms that the holder of government bonds receives each period is set to 0.04. Different values have been tried but do not significantly affect the results. We set the maturity parameter at  $\rho = 0.96$ , equivalent to an average duration of 5 years. As we vary  $\rho$  in one of the experiments, the parameter governing the average duration of government debt, the steady state bond price changes as well. In order for different maturities to be comparable, we must make sure that the fraction of government debt on the balance sheet of the financial intermediaries does not change. Hence we calibrate on the outstanding government liabilities as a fraction of GDP  $\bar{q}_b \bar{b}/\bar{y}$  instead of  $\bar{b}/\bar{y}$ , and set this equal to 3.2, implying an annual debt-to-GDP ratio of 80 percent. Even though government financing by financial intermediaries accounts for only a small part in the US, most financial friction models have been calibrated on US data. At first we follow the conventional calibration, leaving calibration on European data for the future. Even though we will not be able to perform quantitative exercises for the debt-distressed European periphery economies, the current model will be capable to highlight the relevant mechanisms. We assume more aggressive monetary policy in the face of a credit crisis, and hence set  $\rho_r = 0.4$  in times of crisis. We think this captures the way central banks reacted when the financial crisis erupted. We do not set it to zero, in order not to violate the ZLB (zero lower bound). A credit crisis is represented by a negative shock to capital quality  $\xi_t$  of 5 percent on impact, with an autocorrelation coefficient  $\rho_\xi = 0.66$ , as in Gertler and Karadi (2011).

The calibration of the real economy is not affected by the introduction of sovereign default risk. For the financial sector, the steady state bond price  $\bar{q}_b$  changes, and hence  $\bar{b}$ . We calibrate the maximum level of government liabilities  $\bar{q}_b \bar{b}_{max}/\bar{y}$  to be at 90% of annual steady state GDP.

Parameter	Value	Definition
<i>Households</i>		
$\beta$	0.990	Discount rate
$\nu$	0.815	Degree of habit formation
$\Psi$	3.409	Relative utility weight of labor
$\varphi$	0.276	Inverse Frisch elasticity of labor supply
<i>Financial Intermediaries</i>		
$\lambda$	0.3863	Fraction of assets that can be diverted
$\chi$	0.0021	Proportional transfer to entering bankers
$\theta$	0.9722	Survival rate of the bankers
$\Gamma$	0.0025	Steady state credit spread $E[r^k - r^d]$
<i>Intermediate good firms</i>		
$\epsilon$	4.176	Elasticity of substitution
$\psi$	0.779	Calvo probability of keeping prices fixed
$\alpha$	0.330	Effective capital share
<i>Capital good firms</i>		
$\gamma$	1.728	Investment adjustment cost parameter
$\delta$	0.0494	Steady state depreciation rate
<i>Autoregressive components</i>		
$\rho_z$	0.95	Autoregressive component of productivity
$\rho_\xi$	0.66	Autoregressive component of capital quality
$\rho_g$	0.800	Persistence in government spending
$\rho_r$	0.800	Interest rate smoothing parameter
<i>Policy</i>		
$r_c$	0.04	Real payment to government bondholder
$\rho$	0.96	Parameter government debt duration (5 yrs)
$\kappa_b$	0.050	Tax feedback parameter from government debt
$\kappa_\pi$	1.500	Inflation feedback on nominal interest rate
$\kappa_y$	0.125	Output feedback on nominal interest rate
<i>Default parameters</i>		
$\bar{\Delta}$	0.005	Steady state share of default indicator
<i>Shocks</i>		
$\sigma_z$	0.010	Standard deviation productivity shock
$\sigma_\xi$	0.050	Standard deviation capital quality shock
$\sigma_u$	0.050	Standard deviation unannounced government spending shock
$\sigma_a$	0.050	Standard deviation pre-announced government spending shock
$\sigma_r$	0.0025	Standard deviation interest rate surprise shock
<i>Option parameters</i>		
$r$	-0.0273	Compounded risk-free interest rate
$\sigma$	0.5031	Standard deviation underlying process
$T$	0.1107	Time to maturity

Table 2: Table with model parameters.

Different values could have been chosen, but the main point of the paper is to show the mechanisms that interplay when debt levels get close to the maximum level of debt. The steady state fraction of government liabilities  $\bar{q}_b \bar{b} / \bar{y}$  on the balance sheet of financial intermediaries does not change, since it is still calibrated to equal 80% of annualized steady state output. The steady state default probability is set at a rather conservative estimate of  $\bar{\Delta} = 0.005$ , which implies an annual default probability of 2%, which is small given the observed bond spreads in the European periphery.

When far away from the maximum level of debt, we target  $\bar{q}_b \bar{b}$ ,  $\bar{q}_b \bar{b}_{max}$  and  $\bar{\Delta}$  to find the parameters  $r, \sigma$  and  $T$  of the default approximation function (see appendix 9.1). When close to the maximum level of debt, however, this calibration strategy (calibration strategy 1 in the appendix), is not numerically possible anymore. In that case we bring the steady state government liabilities  $\bar{q}_b \bar{b}$  as close as possible to the maximum level of debt while still applying calibration strategy 1 (in this case 75% of annual steady state GDP), and find the accompanying parameters  $r, \sigma$  and  $T$  of the default approximation function. We then employ calibration strategy 2, in which we target  $\bar{q}_b \bar{b}$ ,  $\bar{q}_b \bar{b}_{max}$ , and use the parameters from the default approximation function to back out the steady state default probability to find  $\bar{\Delta} = 0.0068$  (for  $\bar{q}_b \bar{b}$  equal to 80% of annual steady state GDP). This calibration strategy is used in the first 2 experiments.

## 5 Results

In this section we use the model to trace out dynamic multipliers after Keynesian demand stimuli. We start off with an economy that has been shocked by a financial crisis, modeled as a sudden unanticipated downward shock to the value of commercial banks' assets, without any government response. Following Gertler and Karadi (2011), we implement the downward shock to bank assets as a downward shock to the "quality" of capital. We then analyse the output response to an expansionary shift in government expenditure in response to the shock. In the first set of policy experiments, we use the full model, with long term debt held on banks' balance sheets and sovereign risk built in. In line with budget procedures in practice everywhere, we assume that the stimulus package is announced at the onset of the financial crisis, but implemented only four quarters (one budget year) after its announcement. After presenting our core results and the dynamic multiplier patterns we find, we dissect these results by running similar simulations but trimming the model down step by step, so as to find out which new feature is the most significant driver of the strong results we obtain on the size of multipliers. The policy response, which we feed into the model as a second, anticipated shock, is graphically represented in figure 5.

### 5.1 The effects of a delayed stimulus package in the presence of long term debt and sovereign default risk

We start by investigating the effects of a delayed stimulus package that is announced when the crisis (capital quality shock) hits in the initial period. We realistically assume that there is a time delay between the announcement and implementation of the stimulus package equal to one budget cycle (4 quarters). To set the stage, figure 6 compares the stimulus package with the case where sovereign default risk is present, but with no additional government policy being implemented (blue solid line in the figure). The steady state level of government liabilities is at 80% of annual steady state output, with the maximum level of liabilities at 90% of annual steady state output. Looking at the no policy response case, we see that the deterioration in the capital quality  $\xi_t$  induces losses at the financial intermediaries on the private loans: lower than expected returns on the private



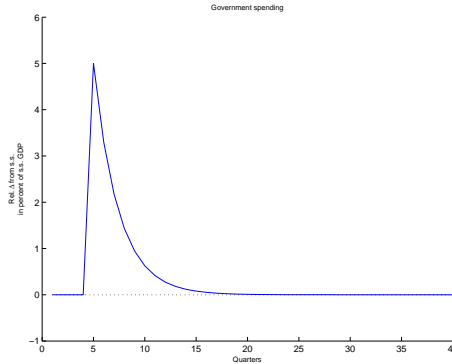


Figure 5: Figure displaying the delayed stimulus spending in case the government engages in additional policy. The size of the stimulus is equal to 5% of quarterly steady state output, or equivalently, 1.25% of annual steady state output.

loans reduce net worth, making the financial intermediaries more balance sheet constrained. The credit spread increases by almost 150 basispoints, thereby increasing interest rates on loans, which reduces demand for loans from intermediate goods producers. Lower demand translates into lower capital prices  $q_t^k$ , further increasing losses, due to lower resale prices for the ‘old’ capital that the intermediate goods producers are selling to the capital producers. Arbitrage between private loans and government bonds causes the returns on bonds to go up and bond prices to fall accordingly. This price fall triggers further capital losses at the financial intermediaries. Because of these additional declines in net worth, a second round of interest rate increases follows, further reducing the demand for capital. Thus several amplification channels operate, with a significant negative impact on the economy: the subsequent rounds of balance sheet deterioration cause investment and capital to drop by more than 10 percent.

Apart from the direct effect of capital losses on private loans and government bonds, investment (and hence capital formation) are negatively affected also because of direct crowding out by government bonds in bank portfolio’s, a crowding out channel highlighted in Kirchner and van Wijnbergen (2012): the government and private enterprise compete for funds that are not perfectly elastically supplied because of the leverage constraints under which the banks have to operate. The direct effect is again higher interest rates and thus falling bond prices, which once again adds an amplification channel: a lower bond price means that the government has to issue more bonds for any *given* revenue raising target. This effect is further amplified because the size of the balance sheet is reduced as well, due to the tightening of the balance sheet constraint. Obviously all these effects are stronger the larger the additional government purchases.

The lower capital quality leads to lower wages, as the marginal product of labor falls with (effectively) less capital at its disposal. Lower wages and reduced profits translate into lower consumption: output and consumption are reduced by more than 4 percent. We see that even after 40 quarters the economy still has not recovered completely from the initial shock. This model therefore highlights the point made by Reinhart and Rogoff (2009): financial crises lead to long lasting declines in output.

Now consider the impact of a government response through a Keynesian stimulus package: due to the forward looking nature of the model, agents immediately anticipate the future debt issuance,

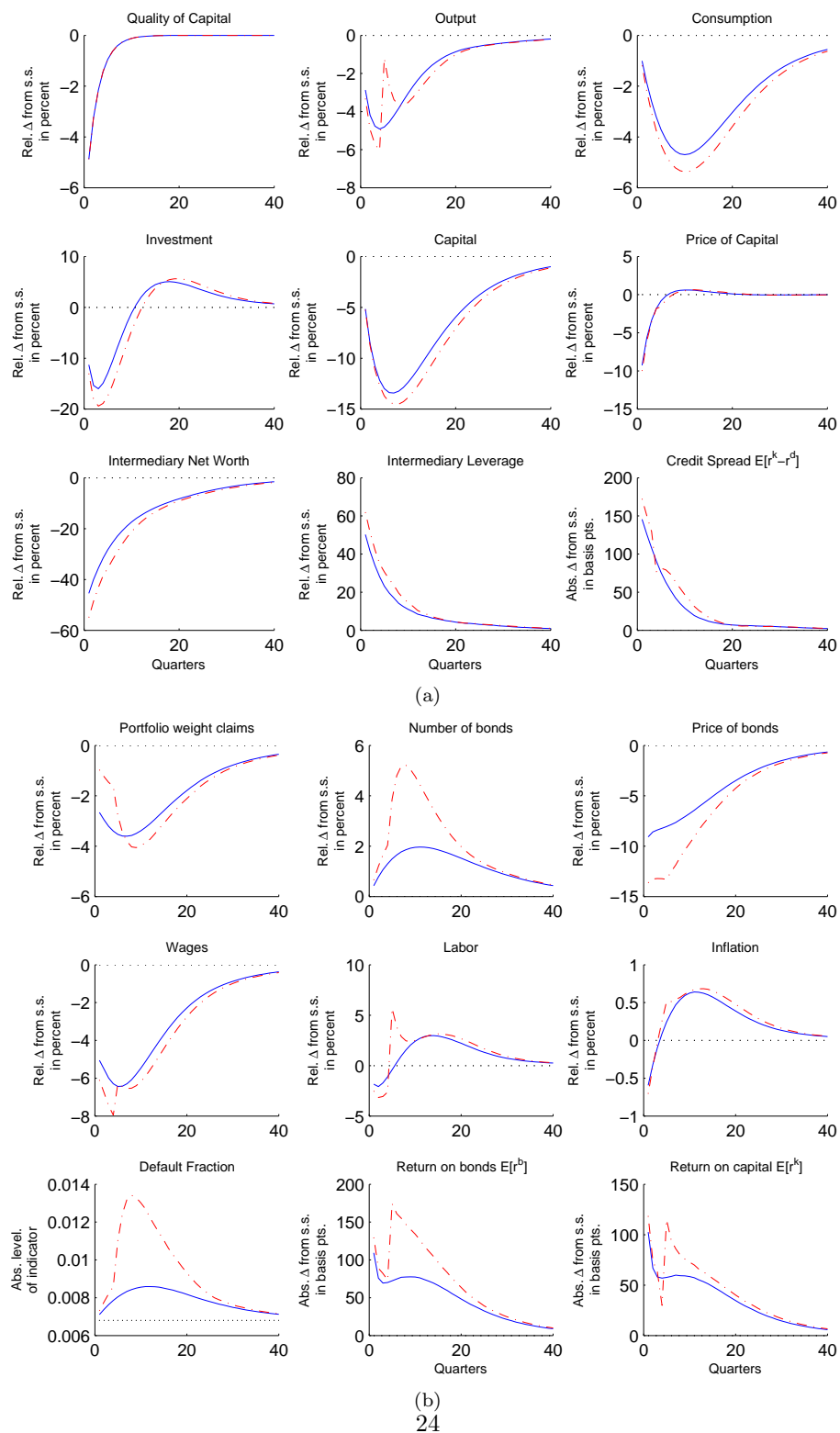


Figure 6: Plot of the impulse response functions comparing no additional policy (blue, solid), and delayed fiscal stimulus (red, slotted), in an economy with sovereign default risk. The delayed stimulus is announced as the crisis hits, and implemented 4 quarters later through additional debt issuance, and equal to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

which will bring the debt level closer to the maximum level of debt, thereby increasing the probability of default. Note that sovereign default risk mostly affects the economy through anticipation effects: the fraction of debt over which the government eventually defaults in this scenario never surpasses the 1.5% (remember that the government partially defaults on a small fraction even before the debt limit is reached because of our approximation of the non-differentiable default function). Hence the effects of sovereign default risk are propagated through a bond price that is lowered ex-ante, increasing capital losses on bank balance sheets, before an actual sovereign default materialises. The anticipation of having to finance riskier debt in the *future* makes the financial intermediaries *immediately* more balance sheet constrained, thereby pushing up the credit spread, increasing (expected) interest rates. Since the cash flow from the government bonds is fixed in real terms, the only way to accommodate a higher interest rate is a lower bond price, which falls by another 5% with respect to the case of no additional government policy. Therefore additional capital losses on government bonds arise, pushing down net worth even more and thus making the intermediaries more balance sheet constrained. This in turn pushes up interest rates further, leverage goes up again and so on. Intermediate goods producers also anticipate future crowding out of private loans by government debt once the stimulus will be implemented, and demand fewer loans, as investment falls due to higher capital costs. Investment falls by almost 5% of steady state investment with respect to the no policy case, with a decline in the capital stock as consequence. This, in turn leads to a fall in potential output. More debt issuance causes households to anticipate higher future taxes, which explains the anticipatory fall in consumption by about 1 percentage point relative to steady state consumption, in order to smooth consumption over time. Despite the positive *direct* effects from higher anticipated government demand in the future, which leads firms to demand more loans everything else remaining equal, the current and anticipated future deterioration in financial conditions trigger a fall in demand for private loans and a fall in investment nevertheless, already *before* implementation of the stimulus. The direct positive impact effects from anticipated government stimulus are apparently not enough to offset the fall in consumption, investment, and output with respect to the no policy case. Due to the temporary nature of the stimulus and the tightened financial conditions, firms decide to service the additional government demand not by accumulating more capital but by relying on an increase in the labor supply, which can be adjusted costlessly every period. The desired increase in the labor supply once the extra government demands needs to be met is brought about by a temporary increase in wages while the additional goods are produced.

Figure 7 shows the time pattern of the Keynesian multiplier, the difference in output between the case of no additional government policy and the one with fiscal stimulus, with and without the dampening features introduced in this paper. The solid (blue) line represents the additional government spending as a percentage of steady state GDP. The (red) slotted line provides the benchmark Keynesian stimulus case without long term debt or sovereign risk and where crowding out in the bank asset portfolio is avoided by tax financing the package. The (black) starred line is our base case, with sovereign risk, long term debt (average maturity 5 years) and deficit financing of the stimulus package.

The anticipation effects from the increased fiscal stimulus and its financing mode clearly cause financial intermediaries to become more balance sheet constrained, pushing down investment even before the package is implemented. Together with higher anticipated taxes, which trigger a fall in consumption, output goes down with respect to the no policy case even in the case without financial frictions and/or capital losses on bank balance sheets (slotted line in Fig. 7). The stimulus itself has a direct positive impact on output, but the immediate multiplier is smaller than 1 while the

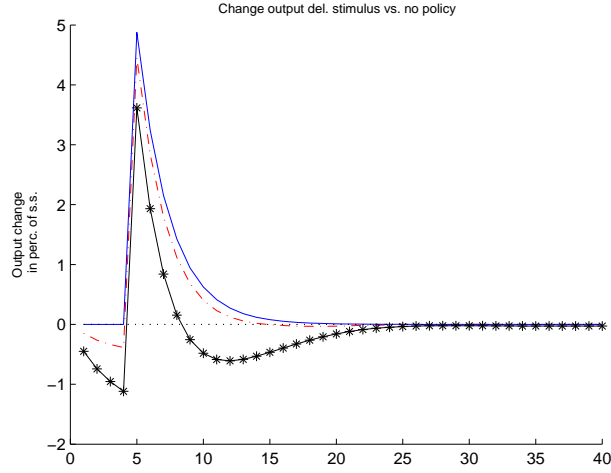


Figure 7: The solid (blue) line represents the fiscal stimulus itself, expressed as a percentage of quarterly steady state output. The slotted and starred lines are fiscal multipliers: the difference between output-with-intervention and output-without-stimulus. The starred (black) line is our base case, with sovereign risk, long term (duration 5 yrs) debt and deficit financed stimulus package. The slotted (red) line is the fiscal multiplier in the benchmark case without sovereign risk, with short term bonds only and with a tax financed stimulus package.

anticipation effects lead to a fall in output before the package is actually implemented. Meanwhile, investment lags the no policy case, thereby reducing the capital stock, and hence potential output. All these effects become considerably worse once the full impact of debt financing on bank balance sheets is introduced, with long term (5 year average maturity) debt and sovereign risk. The initial negative effect is almost three times larger, and the output decline compared to the benchmark case deteriorates to such an extent that the output response actually turns negative one year after the implementation has started and remains below zero for close to six years.

The negative anticipation and crowding out effects via the channels introduced in this paper raise the question of whether the cumulative policy impact can actually turn negative. To answer that question, we calculate a *cumulative discounted multiplier*. Denoting a variable from the stimulus scenario  $x^{st}$  and from the no-policy-response case  $x^{np}$ , the cumulative discounted multiplier is defined as:

$$\mu_D = \frac{\sum_j \beta^j (y_{t+j}^{st} - y_{t+j}^{np})}{\sum_j \beta^j (g_{t+j}^{st} - g_{t+j}^{np})}$$

Table 3 shows the dynamic multiplier for the benchmark case with tax finance, no long term debt and no sovereign risk versus the central case of this paper, where deficits are financed through long term debt, with sovereign risk present and balance sheet constrained intermediaries holding the public debt.

We see that the various negative amplification cycles introduced in this paper have a major impact on the output effects of Keynesian stimulus policies, to the point of that impact being

Stimulus policy	Discounted cumulative multiplier
short term, tax-financed	0.91
long term, sovereign risk	-0.13

Table 3: Table displaying the discounted cumulative multiplier as defined in the text for short term debt and long term financing with default, where steady state government liabilities represent 80% of annual steady state output. Short term debt has an average duration of 2 quarters, whereas average duration of long term debt is equal to 5 years. The multiplier is calculated for the case with no default, except indicated otherwise.

cumulatively negative in discounted value terms. Given the fragile financial environment, with banks intermediating public debt under a leverage constraint, a variety of crowding out channels is at work. Higher debt issuance causes crowding out of private loans by increased levels of government debt as banks switch from corporate loans to government debt to absorb the new debt issued. The long maturity of debt triggers a further fall in the discounted cumulative multiplier, because capital losses on government bonds erode bank capital and lead to a further round of credit tightening. And the increase in sovereign risk associated with the additional debt leads to another round of bank capital losses and credit tightening. As a consequence of all these negative amplification cycles, all related to the intertwinement of banks and sovereigns, the cumulative multiplier turns negative, implying that the stimulus is counterproductive in combatting the recession initiated through the financial crisis. In the next section, we decompose the deterioration in the output effect of the fiscal stimulus and trace it back to the various crowding out channels introduced.

## 5.2 Dissecting the different amplification mechanisms

In this section we look at the different mechanisms that are causing the discounted cumulative multiplier to fall, and eventually even to turn negative. The first channel considered is sovereign default risk, the second channel is through the maturity of the government debt on the balance sheet of the intermediaries, and the third channel is the competition for limited lending capacity between corporates and sovereigns in a leverage constrained financial sector. We show the impact of each of these three crowding out channels in turn.

### 5.2.1 Fiscal stimuli and the effect of sovereign default risk

Consider first the effect of introducing sovereign default risk into the model. The simulations on which we report below show the response to the same financial shock-cum-stimulus package as analysed before, but now comparing the results from one set of runs with debt close to the maximum debt level, where sovereign risk is an issue, with the model response from another set of runs where all assumptions are the same except that the distance to default for the sovereign is much larger so that sovereign risk does not play a role (has no impact on bond prices). The delayed stimulus package is the same as in the previous experiments. The (red) slotted line gives our base case with all the amplification channels in, and the (blue) solid line refer to the case where sovereign risk has been made ineffective by increasing the sovereign's distance to default: when far away from the maximum level of government debt, the additional sovereign default channel does not generate any extra dynamics. All other assumptions and parameters are the same in the two sets. Figure 8 compares a subset of the results for the two cases.

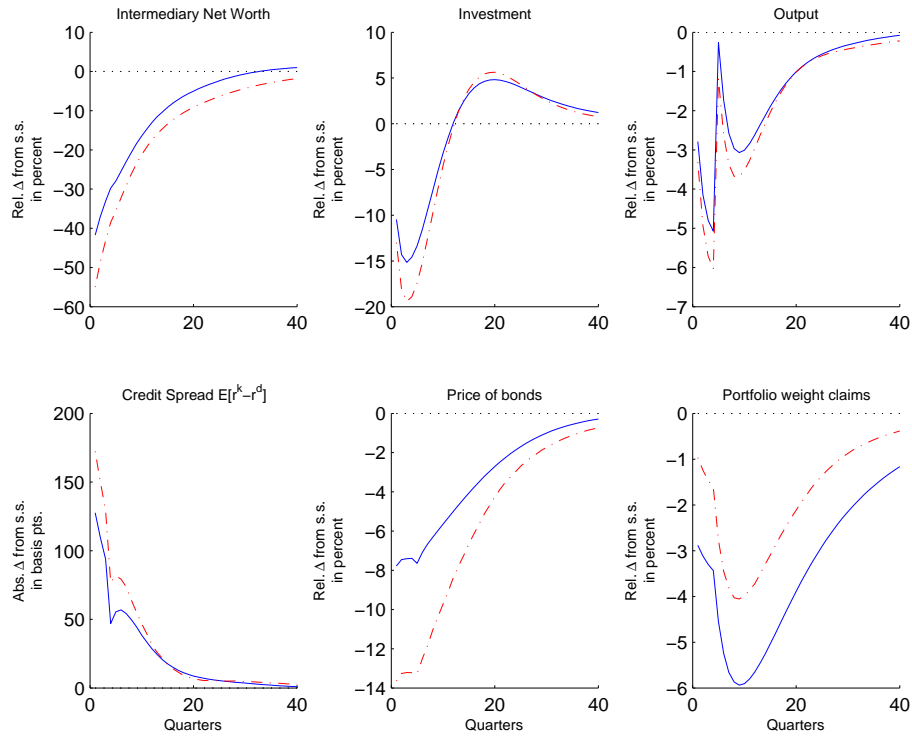


Figure 8: Plot of the impulse response functions comparing the economy with no sovereign default risk (blue, solid), and with sovereign default risk present (red, slotted). In both cases the government engages in preannounced stimulus spending implemented 4 quarters after announcement (at the onset of the crisis) through additional debt issuance. The size of the package amounts to 1.25% of annual steady state GDP. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

Incorporating sovereign default risk in the analysis of a deficit financed fiscal stimulus leads to a significant deterioration of outcomes compared to the no default risk outcome. The credit spread increases (by 50 basispoints), as well as the leverage ratio, since the net worth of the financial intermediaries decreases by an additional 15 percent with respect to the steady state value. The reason that sovereign risk plays such a big role is of course the increase in debt due to the stimulus package, with add on amplification rounds through the impact of bond price declines on later needs to finance and refinance debt, as elaborated upon in the previous section. The initial shock works out the same in both set of runs: due to the fact that the financial intermediaries have to take losses on the private loans because of the initial capital quality shock (the crisis), they become more balance sheet constrained. The leverage constraint becomes more binding, which pushes up (expected) interest rates, reducing the demand for new capital. Due to arbitrage between private loans and bonds, the (expected) return on bonds is driven up as well, driving down the price of bonds. This in turn increases the nominal amount of debt the government needs to issue in order to finance the additional government spending. As a consequence, the government gets closer to the debt limit, increasing the risk of a sovereign default. The financial intermediaries financing the government anticipate the higher default fraction, which leads to a further fall in the bond price, which drops with more than 5 percent on impact compared with the no default case. This in turn, leads to larger additional capital losses for financial intermediaries on their holdings of ‘old’ government bonds. Therefore aggregate financial net worth goes down by more than 15 percent of steady state net worth. The subsequent lower bond price further increases the number of bonds that need to be issued for given revenue targets, moving the debt even closer to the debt limit, and so on.

The impact on the real economy of sovereign risk is substantial. The trough of the recession is half a percentage point deeper, and output remains below the no default case over the entire time path. Due to substantial additional capital losses on government bonds, net worth goes down more, which in turn pushes up (expected) interest rates causing the price of capital to fall further. This has a negative effect on investment, pushing down the capital stock, and thereby pushing down output in the long run. Households become more constrained through lower wages, and lower profits from both the financial and non-financial firms they own, so consumption goes down. Despite the fact that the government eventually defaults over a small fraction of its debt only (less than 1.5%), and the government does not actually surpass the debt limit, the ex-ante pricing effects of the *possibility* of a sovereign default are very substantial, and are largely anticipatory. The fact that the government gets closer to the debt limit, and could go over it if a bad shock arrives, causes investors in government bonds to demand a risk-premium, translating in a lower bond price, where the drop is doubled compared with the no default case. It is clear that the anticipation effect dominates the ex-post effect.

Table 4 shows the impact on the cumulative multiplier  $\mu_D$ . Clearly the emergence of sovereign risk and the associated capital losses on bank holdings of sovereign debt have a major impact: of the total decline in  $\mu_D$  from 0.91 down to -0.13, a full 49% is explained by the capital losses associated with the higher sovereign risk triggered by the debt financed stimulus package.

### 5.2.2 Fiscal stimuli and the maturity of government bonds

Consider next the impact of debt maturity. To bring that out we compare now the same run as the previous section (without sovereign risk but with long term debt) with a set of runs where everything is the same except that the maturity of the debt has been shortened to short term debt

Stimulus policy	Discounted cumulative multiplier
short term, tax-financed	0.91
long term, no sovereign risk	0.38
long term, sovereign risk	-0.13

Table 4: Table displaying the discounted cumulative multiplier as defined in the text for short term debt and long term financing with and without default, where steady state government liabilities represent 80% of annual steady state output. Short term debt has an average duration of 2 quarters, whereas average duration of long term debt is equal to 5 years. The multiplier is calculated for the case with no default, except indicated otherwise.

(two quarters) only (figure 9). The case where all government debt, both old and new, is short term is represented by the solid (blue) line, and the case where 5 year debt (LT) is being used by the slotted (red) line. In both cases, the sovereign default channel has been shut off (by increasing the max debt level sufficiently). Steady state government liabilities are again at 80% of annual steady state output. As before, the fiscal stimulus is announced as the crisis hits, implemented 4 quarters later, and has a size of 1.25% of annual steady state output. The results of the simulations clearly show that lengthening the maturity of the government debt has a strong impact.

Capital losses on government bonds due to falling bond prices are substantially larger in the case of long term debt, by some 5% of their steady state value, so the initial drop in banks' net worth is correspondingly larger and credit tightness as measured by the credit spread goes up more in the long term debt case. This is briefly reversed at the time the package is refinanced, then credit spreads briefly shoot up more in the ST case as a larger package of loans needs to be refinanced in addition to the financing required for the additional expenditure at the very moment credit is tightening anyway.

The increase in the credit spread in the case of long term debt is smaller, but takes much more time to fade away; credit spreads remain higher for much longer in the long term debt case with correspondingly more negative effects on private investment and output. The subsequent lower capital stock reduces potential output and the marginal product of labor, translating into lower wages and labor supply. Together with lower profits, and higher taxes, consumption is pushed down significantly. Lower investment and consumption push down output compared with the case of short-term debt by approximately 1.5 percentage point. The effects are clearly substantial, and work through the channel of larger capital losses on government bonds held on bank balance sheets. Table 5 calculates the impact of the difference in debt maturity on the dynamic multiplier further by adding intermediate rows to the earlier dynamic multiplier table: we now add an intermediate case without sovereign risk and with short term debt (case 2).

Comparing the ST debt case 2 with the LT debt case 3 shows that the dynamic cumulative multiplier is substantially higher in the ST debt case, it increases from 0.38 to 0.75. So of the total difference of 1.04 between the benchmark and the base case, 49% can be attributed to the sovereign risk channel, as seen in the previous subsection, 36% to the impact of longer debt maturity in the base case compared to the benchmark (compare case 2 and 3), with the remaining 15% of the difference (going from 0.75 to 0.91) due to the balance sheet crowding out channel: this refers to the fact that government debt and corporate loans compete for limited lending capacity of the leverage constrained banks (cf Kirchner and van Wijnbergen (2012)).

In figure 10 we plot the decline in the cumulative dynamic multiplier  $\mu_D$  as a function of average



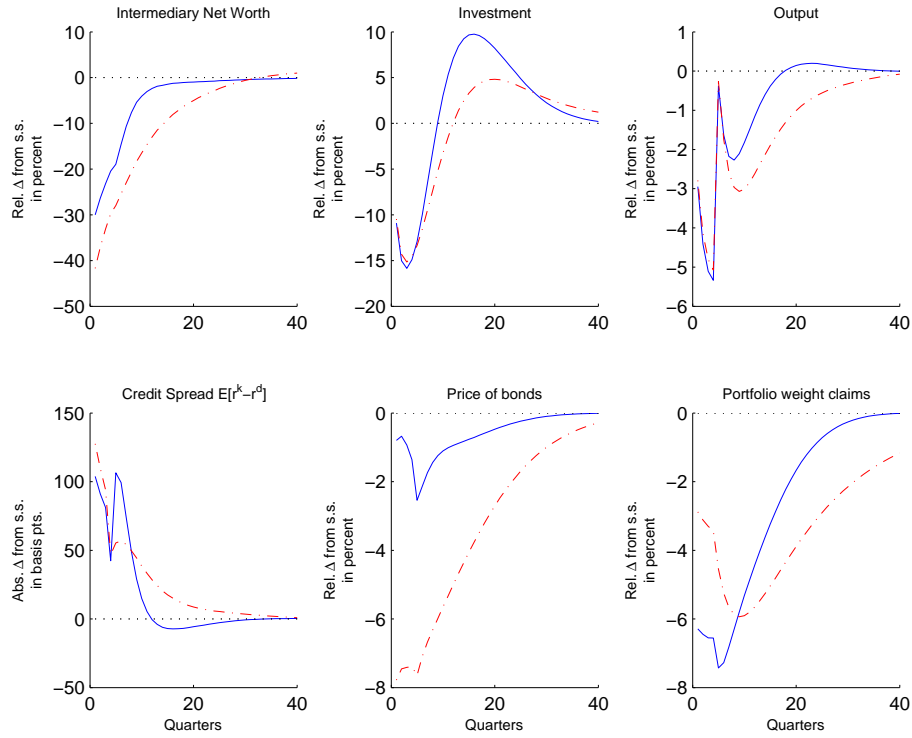


Figure 9: Plot of the impulse response functions for the no default economy comparing impulse responses assuming government debt with average duration of 2 quarters (blue, solid) versus a maturity of 5 years (red, slotted).

Stimulus policy	Discounted cumulative multiplier
1: short term, tax-financed, no sovereign risk	0.91
2: short term, no sovereign risk	0.75
3: long term, no sovereign risk	0.38
4: long term, sovereign risk	-0.13
5: long term (10 yrs.), sovereign risk	-0.37

Table 5: Table displaying the discounted cumulative dynamic multiplier for listed scenarios. In all cases steady state government liabilities represent 80% of annual steady state output. Short term debt has an average maturity of 2 quarters, whereas duration of long term debt is equal to 5 years in case 3 and 4, and 10 years in scenario 5.

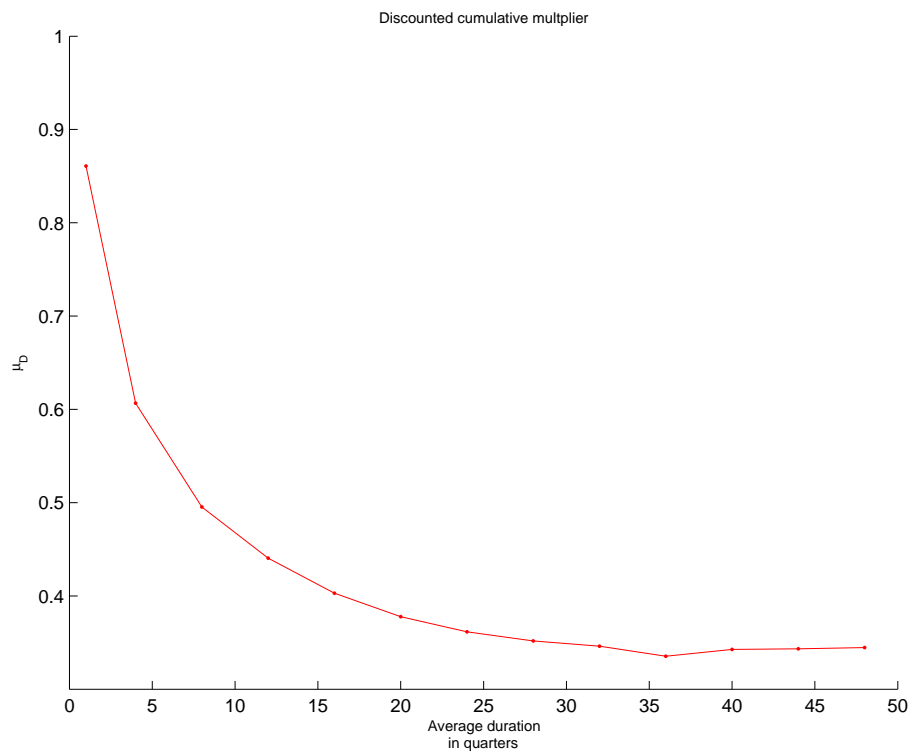


Figure 10: Duration vs. discounted multiplier with steady state government liabilities at 80% of annualized steady state GDP in the no default economy.

debt maturity of existing and new debt (we recalculate  $\rho$  into the more intuitive but equivalent metric of average maturity (duration), measured in quarters). The figure shows that most of the impact happens as the maturity lengthens to 5 years, then the impact stabilizes.

### 5.3 Fiscal stimuli and accommodative monetary policy

If capital losses on outstanding debt are the main driver of negative amplifications after a deficit financed stimulus, it is logical to check whether accommodating monetary policy can alleviate that problem. Capital losses are triggered by falling bond prices, and monetary policy aims to lower interest rates which should to some extent offset this fall. Hence accompanying a deficit financed stimulus with an accommodating monetary shock seems a natural response. We therefore conclude by performing the same deficit financed stimulus package in the economy subject to sovereign default risk as in section 5.1, but now we let monetary policy accommodate the fiscal stimulus package. We adjust the Taylor rule in the following way:

$$r_t^n = (1 - \rho_r)(r^n + \kappa_\pi(\pi_t - \bar{\pi}) + \kappa_y \log(y_t/y_{t-1})) + \rho_r r_{t-1}^n + \kappa_\xi \varepsilon_{\xi,t} + \varepsilon_{r,t}. \quad (35)$$

This implies that when a capital quality shock arrives, the nominal interest rate is adjusted by  $\kappa_\xi \varepsilon_{\xi,t}$  with respect to the regular Taylor rule (in the period the financial crisis hits). We calibrate this term in such a way that when the capital quality shock hits, the nominal interest rate is lowered 100 basispoints everything else equal. The results can be found in figure 11.

It is clear that the monetary stimulus to a substantial extent offsets the negative impact of the various crowding out mechanisms and negative amplification cycles explored so far. In particular the output figure shows that both negative segments of the dynamic output response are less severe. The nominal funding costs of financial intermediaries are lowered because of a drop in the nominal interest rate which after offsetting effects have come into play ends at more than 50 basispoints, about half the size of the initial policy shock. Net worth increases, and the leverage constraint becomes less binding, as can be seen from a decreased credit spread. Both the price of capital and of government bonds increase, lowering (expected) interest rates. This, in turn, increases demand for physical capital, with investment increasing by 5% of the steady state value. Output is lifted across the entire time-path, showing the potency of monetary policy to alleviate the balance sheet constraint of intermediaries by directly lowering the funding costs.

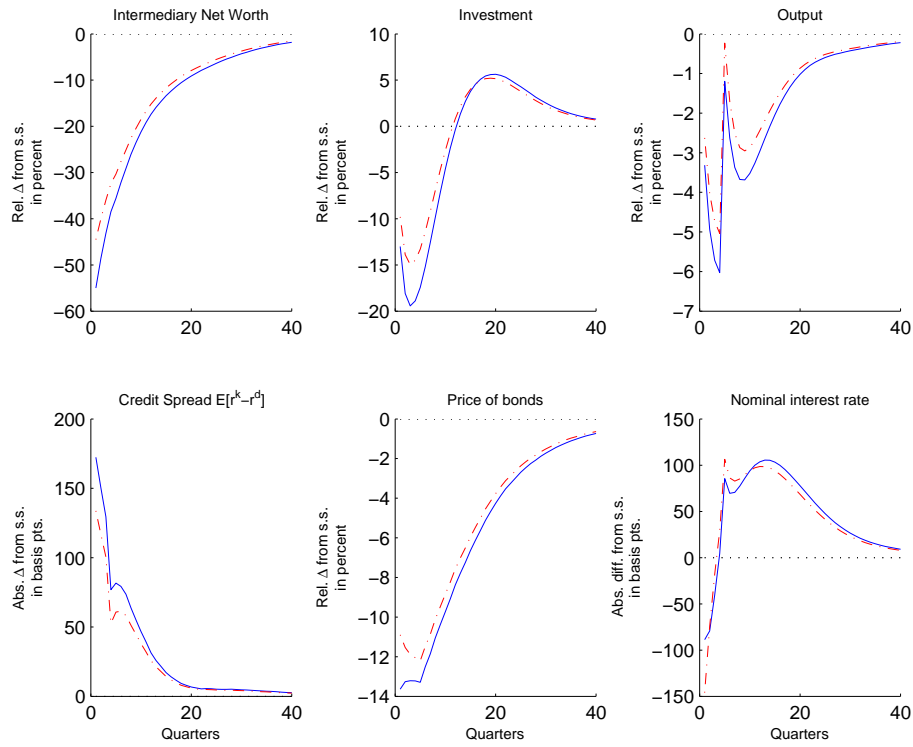


Figure 11: Plot of the impulse response functions for a delayed fiscal stimulus without additional monetary stimulus (solid, blue), and with monetary stimulus (red, slotted), in an economy with sovereign default risk. The delayed stimulus is announced as the crisis hits, and implemented 4 quarters later through additional debt issue, and equal to 1.25% of annual steady state GDP. The monetary stimulus consists of a drop in the nominal interest rate of 100 bps. everything else equal. The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

## 6 Conclusion

We investigate the impact of fiscal stimulus packages in an environment of weakly capitalised banks that play a significant role in the placement of government debt. The analysis is of particular relevance in current day Southern Europe, where banks have significantly expanded the share of their portfolio invested in sovereign debt, increasingly of their “own” sovereign, and where the financial crisis has left banks undercapitalised even 5 years after the crisis spilled over into Western Europe following the bankruptcy of Lehman Brothers. Our framework consists of a mostly standard New-Keynesian DSGE model, supplemented by a financial system, with a focus on interactions between financial frictions, sovereign risk and the financing of government deficits, in a setup similar to Van der Kwaak and Van Wijnbergen (2014). The financial frictions part builds on Gertler and Karadi (2011), like them we introduce leverage constraints arising from asymmetric information problems between depositors and bank managers/shareholders. A key aspect of our model structure is that commercial banks play a role in the financing of government debt; they act as placement agents for sovereign debt, collecting deposits on the one hand and investing the proceeds partially in loans to the government and partially in loans to the private sector. The innovation of the current paper is the interaction between financial fragility and fiscal stimulus. Thus we extend the Gertler-Karadi (2011) approach by introducing asset choice in bankportfolios (banks now both lend to governments and the private sector), long term sovereign debt and sovereign default risk, following a model of sovereign risk pricing introduced in Claessens and van Wijnbergen (1992). The latter two extensions (long term debt and sovereign risk), similar in setup to Van der Kwaak and Van Wijnbergen (2014), introduce a crucial feature of the current euro crisis in the framework: capital losses by commercial banks on their holdings of sovereign debt.

We analyse the realistic case of a stimulus package announced immediately when the financial crisis hits but implemented only four quarters (one budget cycle) later. The average duration of the debt is calibrated to match the typical maturity structure of a periphery eurozone country (5 years). Financial frictions and sovereign debt problems cast their shadows ahead of the implementation, leading to higher interest rates in the run up to the implementation of the package, in the same way unsustainable deficits in countries like Greece immediately led to higher sovereign debt discounts and higher interest rates. In the run up to the implementation, consumption, output and investment go down with respect to the no intervention case, even though future demand is expected to be higher once the spending package is implemented. This is due to the anticipation of the credit tightening that is foreseen once the package is implemented: the announced government spending causes the bond price to go down immediately, because of the upcoming debt issuance. Investors anticipate the increased bond supply, and the fact that this will bring the government closer to the maximum level of debt, a feature we introduce to capture sovereign default risk. Increased sovereign default risk causes investors to demand a higher premium, which translates into a lower bond price, which in turn inflicts further capital losses on ‘old’ bondholders (the banks). Financial intermediaries become more balance sheet constrained, charging higher interest rates, which causes firms to demand fewer loans: investment drops significantly and lowers the capital stock. So short term output gains, if there are any at all, are bought at the expense of future output as potential output growth actually slows down in response to the stimulus measures. The impact (instantaneous) Keynesian multiplier of the package is smaller than 1, but anticipation effects leading up to the package, and credit tightening afterwards actually cause negative output effects of the stimulus before the start of the program and after the initial positive impact: before and after the quarters in which the stimulus measures are initiated, multipliers are actually negative.

Dissecting the results, we find three important channels that cause the effectiveness of fiscal stimulus to be reduced to the point of becoming negative. The first channel is sovereign default risk, which we introduce by assuming that there is a maximum level of government debt that can be serviced. Above the limit, the government defaults in such a way that brings the debt level back to the maximum level of debt. Far away from the debt limit, the sovereign risk channel does not affect the economy. When close to this limit, however, sovereign default risk does have a substantial effect. Although we do not go over this limit in our simulations, the mere *possibility* of a bad shock pushing debt over this limit causes investors to increase interest rates immediately, which translates into lower bond prices, causing additional losses at the ‘old’ bondholders. Financial intermediaries become more balance sheet constrained, opening up a link between sovereign debt problems and financial fragility, through which sovereign debt problems have a negative effect on the corporate sector. We also show that longer maturity debt increases capital losses, and thereby decreases the effectiveness of fiscal stimulus packages: the discounted cumulative multiplier decreases with increasing maturity. We also show that capital losses on government bonds do not affect the economy when financial intermediaries are absent, since households do not act under a leverage constraint. Comparing the case of household financing of government bonds and private loans with the case where financial intermediaries perform this function, economic outcomes deteriorate significantly with financial frictions.

Stresstests by the European Banking Authority in 2011 show that the domestic sovereign debt exposure of periphery banks is more than 150% of Tier-1 capital. Capital losses on long term sovereign debt have played an important role in the current euro crisis limiting the scope for expansionary fiscal policy, since financial sector problems feed into the budgetary problems of sovereigns, which in turn feed back to the financial sector in the form of capital losses on long term sovereign debt holdings. Of course we do not argue that multipliers are always negative; but we show that financial fragility, sovereign risk problems and their interaction may severely lower them, possibly to the point of becoming negative.

The negative balance sheet effects of deficit financed Keynesian stimulus packages are likely to be less if banks are better capitalised; thus a recapitalisation of banks after a financial crisis (as analysed in Van der Kwaak and Van Wijnbergen (2014)) may well provide more room for fiscal stimulus measures. That would also explain why fiscal expansion seems to have worked well in the United States, where financial intermediaries were forced to clean up their balance sheets in 2009, and where banks anyhow play a smaller role in the provision of public and private debt finance. Our model indicates that in Europe, on the contrary, additional government stimulus might backfire, given the weak balance sheets of financial intermediaries and their extensive exposure to their own sovereigns. In spite of extensive government support measures, financial intermediaries still carry substantial unrecognized losses on their balance sheets in the aftermath of the financial crisis (IMF 2011). Thus the financial fragility problem is worse than the data already suggest. Additional government spending is more likely to crowd out private credit in such an environment, and cause further financial intermediary problems because of feedback loops through bond prices, possibly much exacerbated if there is significant sovereign default risk. A more aggressive approach to restoring bank capital ratios might thus widen the scope for expansionary fiscal policy and so promote recovery in debt ridden economies, rather the opposite of what many fear the higher capital ratios envisaged under Basel-III will bring about. In a final experiment we also show that accommodating monetary policy alleviates all the negative crowding out effects and negative amplification cycles that caused the steep decline in the dynamic multiplier, a policy prescription that has been widely followed in practice on both sides of the Atlantic. Similar to our findings,

Brunnermeier and Sannikov (2013) find that contrary to fiscal policy, monetary policy can be an effective instrument in an environment of financial fragility.

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## 8 Appendix-A

### 8.1 Financial intermediaries

The banker’s intermediate period balance sheet is given by:

$$p_{j,t} = n_{j,t} + d_{j,t}$$

The asset side of the bank’s balance sheet has the following structure:

$$p_{j,t} = q_t^k s_{j,t}^k + q_t^b s_{j,t}^b$$

The balance sheet of intermediary  $j$  evolves according to the following law of motion:

$$\begin{aligned} n_{j,t+1} &= (1 + r_{t+1}^k)q_t^k s_{j,t}^k + (1 + r_{t+1}^b)q_t^b s_{j,t}^b - (1 + r_{t+1}^d)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (r_{t+1}^k - r_{t+1}^d)q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d)q_t^b s_{j,t}^b + (1 + r_{t+1}^d)n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t} \end{aligned}$$

The financial intermediary is interested in maximizing expected profits. The banker’s objective is then given by the following recursive optimisation problem:

$$V_{j,t} = \max E_t \left[ \beta \Lambda_{t,t+1} \{ (1 - \theta)n_{j,t+1} + \theta V_{j,t+1} \} \right]$$

where  $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$ . We conjecture the solution to be of the following form, and later check whether this is the case:

$$V_{j,t} = \nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t}$$

We follow Gertler and Karadi (2011) and assume the banker can divert a fraction  $\lambda$  of the assets at the beginning of the period, and transfer these assets costlessly back to the household. This gives rise to the following constraint:

$$V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) \Rightarrow \nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) \quad (36)$$

The optimisation problem can now be formulated in the following way:

$$\max_{\{q_t^k s_{j,t}^k, q_t^b s_{j,t}^b\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b)$$

The Lagrangian for this problem is now given by:

$$\mathcal{L} = (1 + \mu_t)(\nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t}) - \mu_t \lambda (q_t^k s_{j,t}^k + q_t^b s_{j,t}^b)$$

where  $\mu_t$  is the Lagrangian multiplier on the constraint. Hence we get the following first order conditions:

$$\begin{aligned} q_t^k s_{j,t}^k & : (1 + \mu_t)\nu_t^k - \lambda\mu_t = 0 \\ q_t^b s_{j,t}^b & : (1 + \mu_t)\nu_t^b - \lambda\mu_t = 0 \\ \mu_t & : \{\nu_t^k q_t^k s_{j,t}^k + \nu_t^b q_t^b s_{j,t}^b + \eta_t n_{j,t} - \lambda(q_t^k s_{j,t}^k + q_t^b s_{j,t}^b)\}\mu_t = 0 \end{aligned}$$

From the first order conditions we find that  $\nu_t^b = \nu_t^k$ . Hence the leverage constraint (36) can be rewritten in the following way:

$$\nu_t^k (q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) + \eta_t n_{j,t} \geq \lambda (q_t^k s_{j,t}^k + q_t^b s_{j,t}^b) \Rightarrow q_t^k s_{j,t}^k + q_t^b s_{j,t}^b \leq \phi_t n_{j,t}, \quad \phi_t = \frac{\eta_t}{\lambda - \nu_t^k} \quad (37)$$

where  $\phi_t$  denotes the ratio of assets to net worth, which can be seen as the leverage constraint of the financial intermediary. Substitution of the conjectured solution into the right hand side of the Bellman equation gives the following expression for the continuation value of the financial intermediary:

$$\begin{aligned} V_{j,t} & = E_t \left[ \beta \Lambda_{t,t+1} \{ (1 - \theta) n_{j,t+1} + \theta V_{j,t+1} \} \right] = E_t \left[ \Omega_{t+1} n_{j,t+1} \right], \\ \Omega_{t+1} & = \beta \Lambda_{t,t+1} \{ (1 - \theta) + \theta [\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}] \} \end{aligned}$$

$\Omega_{t+1}$  can be thought of as a stochastic discount factor that incorporates the financial friction. Now we can substitute the expression for next period's net worth into the expression above:

$$\begin{aligned} V_{j,t} & = E_t \left[ \Omega_{t+1} n_{j,t+1} \right] = E_t \left[ \Omega_{t+1} \{ (1 + r_{t+1}^k) q_t^k s_{j,t}^k + (1 + r_{t+1}^b) q_t^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \} \right] \\ & = E_t \left[ \Omega_{t+1} \{ (r_{t+1}^k - r_{t+1}^d) q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d) q_t^b s_{j,t}^b + (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n) n_{j,t} \} \right] \quad (38) \end{aligned}$$

After combining the conjectured solution with (38), we find the following first order conditions:

$$\eta_t = E_t \left[ \Omega_{t+1} (1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n) \right] \quad (39)$$

$$\nu_t^k = E_t \left[ \Omega_{t+1} (r_{t+1}^k - r_{t+1}^d) \right] \quad (40)$$

$$\nu_t^b = \nu_t^k = E_t \left[ \Omega_{t+1} (r_{t+1}^b - r_{t+1}^d) \right] \quad (41)$$

$$\Omega_{t+1} = \beta \Lambda_{t,t+1} \{ (1 - \theta) + \theta [\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}] \}$$

## 8.2 Production side

### 8.2.1 Retail firms

The relevant part of the optimization problem of the typical retail firm is now given by:

$$\max_{P_{f,t}} E_t \left[ \sum_{s=0}^{\infty} (\beta \psi)^s \Lambda_{t,t+s} (1/P_{t+s}) [P_{f,t} - P_{t+s}^m] y_{f,t+s} \right]$$

where  $y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} y_t$  is the demand function.  $y_t$  is the output of the final good producing firms, and  $P_t$  the general price level. The expression for the demand function for the retail firms

products will be derived in the next section. Since all the retail firms have access to the same technology, all the firms that are allowed to reset their prices will choose the same new price ( $P_t^*$ ) for their goods. We remember that the relative price  $m_t$  is equal to  $m_t = P_t^m/P_t$ . Differentiation with respect to  $P_{f,t}$  gives the first order condition for the price the retail firms will charge for their products:

$$\frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\psi)^s \lambda_{t+s} P_{t+s}^{\epsilon} P_t^{-\epsilon} m_{t+s} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\psi)^s \lambda_{t+s} P_{t+s}^{\epsilon-1} P_t^{1-\epsilon} y_{t+s}}$$

We define the relative price of the firms that are allowed to reset their prices to be equal to  $\pi_t^* = P_t^*/P_t$ , while gross inflation is defined to be equal to  $\pi_t = P_t/P_{t-1}$ . The above first order condition can now be rewritten in the following form:

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\Xi_{1,t}}{\Xi_{2,t}} \quad (42)$$

$$\Xi_{1,t} = \lambda_t m_t y_t + \beta\psi E_t \pi_{t+1}^{\epsilon} \Xi_{1,t+1} \quad (43)$$

$$\Xi_{2,t} = \lambda_t y_t + \beta\psi E_t \pi_{t+1}^{\epsilon-1} \Xi_{2,t+1} \quad (44)$$

The aggregate price level equals:

$$P_t^{1-\epsilon} = (1 - \psi)(P_t^*)^{1-\epsilon} + \psi P_{t-1}^{1-\epsilon}$$

The aggregate price level will be given by the following law of motion:

$$(1 - \psi)(\pi_t^*)^{1-\epsilon} + \psi \pi_t^{\epsilon-1} = 1 \quad (45)$$

### 8.2.2 Aggregation

First recall that  $y_{f,t} = y_{i,t} = y_t (P_{f,t}/P_t)^{-\epsilon}$ , for all  $f$  and  $i$ . Hence we can write the factor demands by firm  $i$  as:

$$h_{i,t} = (1 - \alpha)m_t y_{f,t}/w_t, \quad k_{i,t-1} = \alpha m_t y_{f,t} / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta)\xi_t]$$

Aggregation over all firms  $i$  gives us aggregate labor and capital:

$$h_t = (1 - \alpha)m_t y_t \mathcal{D}_t / w_t, \quad k_{t-1} = \alpha m_t y_t \mathcal{D}_t / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta)\xi_t]$$

where  $\mathcal{D}_t = \int_0^1 (P_{f,t}/P_t)^{-\epsilon} df$  denotes the price dispersion. It is given by the following recursive form:

$$\mathcal{D}_t = (1 - \psi)(\pi_t^*)^{-\epsilon} + \psi \pi_t^{\epsilon} \mathcal{D}_{t-1} \quad (46)$$

Now we calculate the aggregate capital-labor ratio, and see that it is equal to the individual capital-labor ratio:

$$k_{t-1}/h_t = \alpha(1 - \alpha)^{-1} w_t / [q_{t-1}^k (1 + r_t^k) - q_t^k (1 - \delta)\xi_t] = k_{i,t-1}/h_{i,t} \quad (47)$$

Calculate aggregate supply by aggregating  $y_{i,t} = a_t (\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha}$ :

$$\int_0^1 a_t (\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha} di = a_t \xi_t^{\alpha} \left( \frac{k_{t-1}}{h_t} \right)^{\alpha} \int_0^1 h_{i,t} di = a_t (\xi_t k_{t-1})^{\alpha} h_t^{1-\alpha}$$

while aggregation over  $y_{i,t}$  gives:

$$\int_0^1 y_{i,t} df = y_t \int_0^1 (P_{f,t}/P_t)^{-\epsilon} df = y_t \mathcal{D}_t$$

Hence we get the following relation for aggregate supply  $y_t$ :

$$y_t \mathcal{D}_t = a_t (\xi_t k_{t-1})^\alpha h_t^{1-\alpha} \quad (48)$$

### 8.3 Derivation of structural equations financial intermediaries in presence of sovereign default risk

The introduction of sovereign default risk changes the equations for the financial intermediaries. In this section we show that the net worth of an individual intermediary is given by the same expression as the case with no default, except for the fact that we replace  $r_t^b$  by  $r_t^{b*}$ . Hence we only have to replace  $r_t^b$  in the equations governing the financial intermediaries by  $r_t^{b*}$ , and include the expression for  $r_t^{b*}$  in the first order conditions. We start by observing that the funds obtained from selling the bonds in period  $t+1$ , that were purchased in period  $t$ , are reduced by  $1 - \Delta_{t+1}$ , just as the fixed real payment  $r_c$  per bond is reduced to  $(1 - \Delta_{t+1})r_c$ . Hence the law of motion for the net worth of an individual intermediary changes into the following equation:

$$\begin{aligned} n_{j,t+1} &= (1 + r_{t+1}^k) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) r_c s_{j,t}^b + (1 - \Delta_{t+1}) \rho q_{t+1}^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (1 + r_{t+1}^k) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) (r_c \rho q_{t+1}^b) s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (1 + r_{t+1}^k) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) \frac{(r_c \rho q_{t+1}^b)}{q_t^b} q_t^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (1 + r_{t+1}^k) q_t^k s_{j,t}^k + (1 - \Delta_{t+1}) (1 + r_{t+1}^b) q_t^b s_{j,t}^b - (1 + r_{t+1}^d) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (r_{t+1}^k - r_{t+1}^d) q_t^k s_{j,t}^k + (r_{t+1}^{b*} - r_{t+1}^d) q_t^b s_{j,t}^b + (1 + r_{t+1}^d) n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t} \end{aligned}$$

where  $r_t^{b*}$  is given by:

$$1 + r_t^{b*} = (1 - \Delta_t) (1 + r_t^b) = (1 - \Delta_t) \left( \frac{r_c + \rho q_t^b}{q_{t-1}^b} \right)$$

We replace  $r_t^b$  by  $r_t^{b*}$  in the equation for the shadow value of government bonds, and the law of motion of net worth:

$$\begin{aligned} \nu_t^b &= E_t \left[ \Omega_{t+1} (r_{t+1}^{b*} - r_{t+1}^d) \right] \\ n_t &= \theta \left[ (r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^{b*} - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \right] + \chi p_{t-1} + n_t^g - \tilde{n}_t^g \end{aligned}$$

The other equations for the financial intermediaries remain the same.

## 9 Appendix-B

### 9.1 Approximation of the default function

We can also write the debt level structure (29) in the following way:

$$b_t = \min \left( \tilde{b}_t, b_t^{max} \right) = b_t^{max} - \max \left( b_t^{max} - \tilde{b}_t, 0 \right) \quad (49)$$

We can interpret the second term of the new debt level as the payoff of a put option at maturity with underlying process  $\tilde{b}_t$  and strike price  $b_t^{max}$ . The formula for  $b_t$ , however, does not have a defined derivative at  $\tilde{b}_t = b_t^{max}$ . Therefore we apply an approximation for the payoff structure of the put option, and use the option pricing formula, which gives the price of the put option when time to maturity is equal to  $T$ , compounded risk-free interest rate  $r$ , and volatility of the underlying process  $\sigma$ . This is an *approximation* to the actual mapping from  $\tilde{b}_t$  to  $b_t$ , and has in this sense no economic interpretation in our model. The red line in figure 12 is the approximation to the *actual* mapping of  $\tilde{b}_t$  to  $b_t$ . We then get the following approximation for  $b_t$ , with  $\Phi(\cdot)$  denoting the standard normal CDF, which is indeed continuous:

$$b_t = b_t^{max} - put_t \quad (50)$$

$$put_t = Xe^{-rT}\Phi(-d_{2,t}) - S_t\Phi(-d_{1,t}) \quad (51)$$

$$d_{1,t} = \frac{\log(S/X) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (52)$$

$$d_{2,t} = \frac{\log(S/X) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (53)$$

$$X = b_t^{max} \quad (54)$$

$$S_t = \tilde{b}_t \quad (55)$$

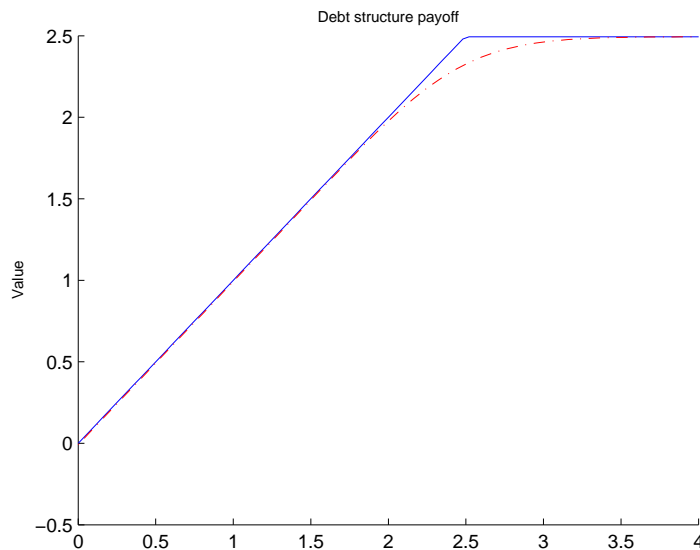


Figure 12: Plot showing the mapping from the no default level of debt  $\tilde{b}_t$  to the actual debt level  $b_t$ . The blue line is the actual mapping, while the red line is an approximation to it, where option pricing formulas have been used.

## 9.2 Calibration strategies

In this section we will write down the 2 calibration strategies regarding the sovereign debt in the current paper, since other parts of the model are straightforward to calibrate. We have the following 2 equations from the financial intermediaries' problem, from which we can derive the steady state return on bonds ex-post a possible default.

$$\begin{aligned}\nu_t^k &= E_t \left[ \Omega_{t+1} (r_{t+1}^k - r_{t+1}^d) \right] \\ \nu_t^b &= E_t \left[ \Omega_{t+1} (r_{t+1}^{b*} - r_{t+1}^d) \right] \\ \Rightarrow E_t \left[ \Omega_{t+1} (r_{t+1}^k - r_{t+1}^{b*}) \right] &= 0\end{aligned}$$

From these equations it is clear that  $\bar{r}_k = \bar{r}_{b^*}$ . Now we have the following equation for the maximum level of debt:

$$b_t^{max} = \bar{b} + \frac{E_t [\tau_{t+1}^{max}] - \bar{\tau}}{\kappa_b} \quad (56)$$

The government budget constraint in case of no default by the government is equal to:

$$\begin{aligned}q_t^b \tilde{b}_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (r_c + \rho q_t^b) b_{t-1} \\ &= g_t + n_t^g + \frac{(r_c + \rho q_t^b)}{q_{t-1}^b} q_{t-1}^b b_{t-1} \Rightarrow \\ q_t^b \tilde{b}_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (1 + r_t^b) q_{t-1}^b b_{t-1}\end{aligned} \quad (57)$$

The mapping from the number of no default bonds to the actual number of bonds is given by:

$$b_t = b_t^{max} - \max(b_t^{max} - \tilde{b}_t, 0) \approx b_t^{max} - put(b_t^{max}, \tilde{b}_t) \quad (58)$$

The actual number of government bonds is given by:

$$\begin{aligned}q_t^b b_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (1 - \Delta_t) (r_c + \rho q_t^b) b_{t-1} \\ &= g_t + n_t^g + (1 - \Delta_t) \frac{(r_c + \rho q_t^b)}{q_{t-1}^b} q_{t-1}^b b_{t-1} \\ &= g_t + n_t^g + (1 - \Delta_t) (1 + r_t^b) q_{t-1}^b b_{t-1} \Rightarrow \\ q_t^b b_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (1 + r_t^{b*}) q_{t-1}^b b_{t-1}\end{aligned} \quad (59)$$

together with the ex-post default return on bonds:

$$1 + r_t^{b*} = (1 - \Delta_t) (1 + r_t^b) \quad (60)$$

and the return on bonds before default:

$$1 + r_t^b = \frac{(r_c + \rho q_t^b)}{q_{t-1}^b} \quad (61)$$

Throughout the paper we assume that the financial intermediaries do not receive any support from the government in the steady state, nor are they paying back support in the steady state, i.e.  $\bar{n}_g = \tilde{n}_g = 0$ . Finally, we have the option pricing formulas:

$$put_t = b_t^{max} e^{-rT} \Phi(-d_{2,t}) - \tilde{b}_t \Phi(-d_{1,t}) \quad (62)$$

$$d_{1,t} = \frac{\log(\tilde{b}_t/b_t^{max}) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \quad (63)$$

$$d_{2,t} = \frac{\log(\tilde{b}_t/b_t^{max}) + \left(r - \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \quad (64)$$

### Calibration strategy 1

The first strategy targets  $\bar{q}_b \bar{b}$ ,  $\bar{q}_b \bar{b}_{max}$  and  $\bar{\Delta}$ , for which we take 60%, respectively 90% of annual GDP and  $\bar{\Delta} = 0.005$ . Since we know  $\bar{q}_b \bar{b}$ , the steady state return on bonds after default  $\bar{r}_{b*}$  and  $\bar{g}$ , we can find the steady state level of taxes from (59):

$$\begin{aligned} \bar{q}_b \bar{b} + \bar{\tau} &= \bar{g} + (1 + \bar{r}_{b*}) \bar{q}_b \bar{b} \implies \\ \bar{\tau} &= \bar{g} + \bar{r}_{b*} \bar{q}_b \bar{b} \end{aligned}$$

Since we know the steady state default fraction  $\bar{\Delta}$  and the ex-post return on bonds, we can calculate  $\bar{r}_b$  through (60):

$$\begin{aligned} 1 + \bar{r}_{b*} &= (1 - \bar{\Delta})(1 + \bar{r}_b) \implies \\ \bar{r}_b &= \frac{1 + \bar{r}_{b*}}{1 - \bar{\Delta}} - 1 \end{aligned}$$

Since we know  $r_c$ , we can find the steady state bond price through (61):

$$\begin{aligned} 1 + \bar{r}_b &= \frac{r_c + \rho \bar{q}_b}{\bar{q}_b} \implies \bar{q}_b (1 + \bar{r}_b) = r_c + \rho \bar{q}_b \implies \\ \bar{q}_b &= \frac{r_c}{1 + \bar{r}_b - \rho} \end{aligned}$$

Now that the steady state bond price is known, we can find the steady state number of bonds and the maximum number of bonds  $\bar{b}$  and  $\bar{b}_{max}$ . Since we know the return on bonds  $\bar{r}_b$ , we can find the number of bonds if the government does not default:

$$\bar{q}_b \bar{b} + \bar{\tau} = \bar{g} + (1 + \bar{r}_b) \bar{q}_b \bar{b}$$

Now that we have found steady state number of bonds  $\bar{b}$ , maximum number of bonds in the steady state  $\bar{b}_{max}$ , and the steady state number of bonds in case the government does not default  $\tilde{b}$ . With these 3 numbers, and the requirement that the derivative of the put option  $-\Phi(-d_{1,t})$  is equal to  $-0.99$ , we can find the variables  $r, \sigma$  and  $T$  from the option pricing formulas.



## Calibration strategy 2

The second strategy targets  $\bar{q}_b \bar{b}$ , and  $\bar{q}_b \bar{b}_{max}$ , and takes the option pricing parameters  $r, \sigma$  and  $T$  as given. We calibrate  $\bar{q}_b \bar{b}$  to be equal to 80% of annual GDP, while  $\bar{q}_b \bar{b}_{max}$  is equal to 90% of annual GDP. Since  $\bar{g}$  is also known, we can find the steady state level of taxes from (59):

$$\begin{aligned}\bar{q}_b \bar{b} + \bar{\tau} &= \bar{g} + (1 + \bar{r}_{b*}) \bar{q}_b \bar{b} \implies \\ \bar{\tau} &= \bar{g} + \bar{r}_{b*} \bar{q}_b \bar{b}\end{aligned}$$

Since we know  $\bar{q}_b \bar{b}$  and  $\bar{q}_b \bar{b}_{max}$ , we can divide the two to find the ratio  $\bar{b}_{max}/\bar{b}$ . Now we look at the option pricing formulas, and remember that the parameters  $r, \sigma$  and  $T$  are given. We rewrite the put option in the following way:

$$\begin{aligned}b_t &= b_t^{max} - put_t = b_t^{max} - \left\{ b_t^{max} e^{-rT} \Phi(-d_{2,t}) - \tilde{b}_t \Phi(-d_{1,t}) \right\} \\ &= b_t^{max} - b_t^{max} e^{-rT} \Phi(-d_{2,t}) + \tilde{b}_t \Phi(-d_{1,t})\end{aligned}$$

Division by  $b_t$  gives the following expression:

$$1 = \frac{b_t^{max}}{b_t} - \frac{b_t^{max}}{b_t} e^{-rT} \Phi(-d_{2,t}) + \frac{\tilde{b}_t}{b_t} \Phi(-d_{1,t}) \quad (65)$$

Now we look at the formula for  $d_{1,t}$ :

$$\begin{aligned}d_{1,t} &= \frac{\log(\tilde{b}_t/b_t^{max}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \\ &= \frac{\log(\frac{\tilde{b}_t}{b_t} \frac{b_t}{b_t^{max}}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = f\left(\frac{\tilde{b}_t}{b_t}, \frac{b_t^{max}}{b_t}\right)\end{aligned} \quad (66)$$

similarly we find for  $d_{2,t}$ :

$$\begin{aligned}d_{2,t} &= \frac{\log(\tilde{b}_t/b_t^{max}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \\ &= \frac{\log(\frac{\tilde{b}_t}{b_t} \frac{b_t}{b_t^{max}}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = g\left(\frac{\tilde{b}_t}{b_t}, \frac{b_t^{max}}{b_t}\right)\end{aligned} \quad (67)$$

Hence we see that equations (65), (66) and (67) only depend on the ratios  $b_t^{max}/b_t$  and  $\tilde{b}_t/b_t$ . We know the first ratio in steady state, so we can solve for the steady state ratio  $\bar{b}/\bar{b}$ . We also see that regarding the default function, it does not matter whether we calibrate on the number of bonds  $b_t$  or the value of government liabilities  $q_t^b b_t$ , since the bond *ratios* are the variables that matter. Now we can find  $\bar{b}/\bar{b}$ , and hence we find  $\bar{q}_b \bar{b} = \bar{q}_b \bar{b} (\bar{b}/\bar{b})$ . Since we know  $\bar{q}_b \bar{b}$ , we can find  $\bar{r}_b$  from (57):

$$\begin{aligned}\bar{q}_b \bar{b} + \bar{\tau} &= \bar{g} + (1 + \bar{r}_b) \bar{q}_b \bar{b} \implies \\ \bar{r}_b &= \frac{\bar{q}_b \bar{b} + \bar{\tau} - \bar{g}}{\bar{q}_b \bar{b}} - 1\end{aligned}$$

Now we can find the steady state bond price from (61):

$$\begin{aligned}1 + \bar{r}_b &= \frac{r_c + \rho \bar{q}_b}{\bar{q}_b} \implies \bar{q}_b (1 + \bar{r}_b) = r_c + \rho \bar{q}_b \implies \\ \bar{q}_b &= \frac{r_c}{1 + \bar{r}_b - \rho}\end{aligned}$$

after which we know  $\bar{b}$ ,  $\bar{b}_{max}$  and  $\bar{\delta}$ . From (60) we can find the steady state default probability:

$$\begin{aligned}1 + \bar{r}_{b*} &= (1 - \bar{\Delta}) (1 + \bar{r}_b) \implies \\ \bar{\Delta} &= 1 - \frac{1 + \bar{r}_{b*}}{1 + \bar{r}_b}\end{aligned}$$